Mathematica Version Number:4.0.1.0
Platform: Windows 7 version 6.1 (Build 7601: ServicePack 1)

Copy ConvexHull3D.m to your DiscreteMath folder

```
<< DiscreteMath`ConvexHull3D`
```

or to a temporary system folder:

```
In[1]:= << "c:/_Wouter/it/ConvexHull3D.m"
```

Mean Hull Width : reference D. A. Klain and G.-C. Rota, Introduction to Geometric Probability, Cambridge Univ. Press, 1997, pp. 139-140 (as suggested for inclusion by Steven Finch)
Any errors are however mine only (Wouter Meeussen 07/11/2012).

for real input :

```
In[2]:= ps = Table[Random[Real, {-10, 10}],
          {12}, {3}]

        Out[2]= {{-4.86403, 5.25994, 6.11884},
                 {1.57205, -1.09067, 4.45422},
                 {9.92145, -8.19582, -5.16621},
                 {8.25819, -6.78511, 5.11827},
                 {4.55017, -6.43323, 8.80229},
                 {1.40008, -9.66, -4.72616},
                 {-3.13703, 3.2106, 7.57285},
                 {3.22026, -1.95142, -6.25204},
                 {2.43688, 7.96032, 1.92974},
                 {2.17591, -6.47244, -6.49391},
                 {2.00829, 0.371732, 8.69377},
                 {-4.7521, -1.20659, 5.25346}}

        In[3]:= Timing[ch3D = ConvexHull3D[ps]]

        Out[3]= {0.015 Second,
                 {{1, 7, 9}, {1, 8, 10}, {1, 9, 8}, {1, 10, 12},
                  {1, 12, 7}, {3, 4, 6}, {3, 6, 10}, {3, 8, 9},
                  {3, 9, 4}, {3, 10, 8}, {4, 5, 6}, {4, 9, 11},
                  {4, 11, 5}, {5, 7, 12}, {5, 11, 7},
                  {5, 12, 6}, {6, 12, 10}, {7, 11, 9}}}

        In[4]:= HullVolume[ps, ch3D]

        Out[4]= 1107.33

        In[5]:= HullArea[ps, ch3D]

        Out[5]= 642.766

        In[6]:= HullMeanWidth[ps, ch3D]

        Out[6]= 16.0961

        In[7]:= ?*Hull*

ConvexHull3D   HullArea      HullMeanWidth HullShow      HullVolume
```

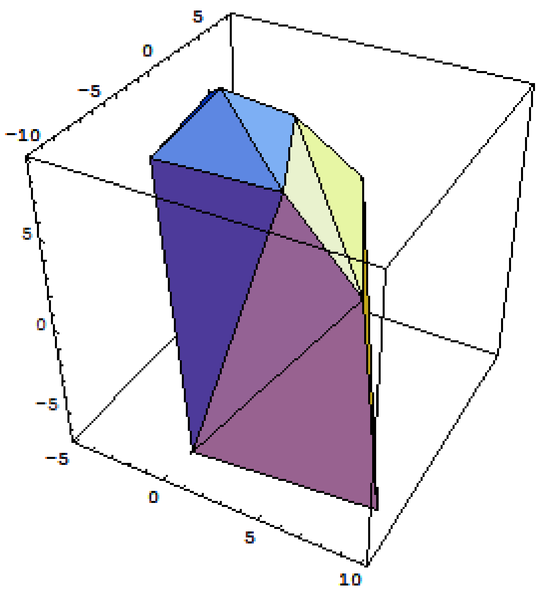
In[8]:= ?dihedralAngle

```
dihedralAngle pointlist:{{x,y,z}..},
 hull,{face_i,face_j}] gives the angle
 between normals on face_i and face_j,
 with face_i the i-the face in hull.
```

In[9]:= ?edgelength

```
edgelength[{i,j}, hull] gives the distance
  between points with indices i and j.
```

In[10]:= HullShow[ps, ch3D]



Out[10]= - Graphics3D -

for integer input :

make sure that your input list is unique : repeat points are not allowed.

```
In[11]:= uniq[li_List] :=
       Block[{f}, f[g_] := (f[g] = Sequence[]; g);
        f /@ li];
```

```
In[12]:= (ps =
     uniq @
      Table[Random[Integer, {-10, 10}],
       {200}, {3}]) // Shallow
```

Out[12]//Shallow=

```
{{-6, 5, 9}, {-10, 7, 9},
 {-4, 1, -7}, {3, 6, -8}, {-5, -10, -9},
 {8, 3, -6}, {-5, -4, -5}, {0, -9, -5},
 {-5, 4, 7}, {-2, -9, -7}, «187»}
```

In[13]:= Timing[ch3D = ConvexHull3D[ps]]

```
Out[13]= {0.468 Second,
  {{2, 40, 146}, {2, 66, 175}, {2, 69, 119},
   {2, 106, 150}, {2, 119, 40}, {2, 146, 106},
   {2, 150, 66}, {2, 175, 174}, {5, 39, 183},
   {5, 40, 119}, {5, 109, 188}, {5, 119, 39},
   {5, 183, 109}, {5, 188, 137}, {16, 112, 69},
   {16, 174, 175}, {16, 175, 183},
   {16, 183, 112}, {35, 59, 144},
   {35, 123, 59}, {35, 137, 188}, {35, 144, 40},
   {35, 188, 123}, {39, 112, 183},
   {39, 119, 112}, {55, 59, 123}, {55, 78, 146},
   {55, 123, 78}, {55, 135, 59}, {59, 135, 144},
   {66, 150, 170}, {66, 170, 175},
   {69, 112, 119}, {73, 109, 183},
   {73, 175, 193}, {73, 183, 175},
   {76, 132, 170}, {76, 156, 132},
   {78, 79, 132}, {78, 106, 146}, {78, 123, 79},
   {78, 132, 156}, {78, 156, 106},
   {79, 84, 132}, {79, 123, 84}, {84, 123, 130},
   {84, 130, 188}, {84, 188, 132},
   {109, 132, 188}, {123, 188, 130},
   {132, 193, 170}, {170, 193, 175},
   {2, 174, 16, 69}, {5, 137, 35, 40},
   {73, 193, 132, 109}, {40, 144, 135, 55, 146},
   {76, 170, 150, 106, 156}}}
```

for integer input, the hull volume is of the form (Integer) / 6

```
In[14]:= HullVolume[ps, ch3D]
         % // N
```

Out[14]= $\frac{41513}{6}$

Out[15]= 6918.83

```
In[16]:= HullArea[ps, ch3D] // Short
         N[%, 24]
```

Out[16]//Short=

$$\frac{1}{2}\left(1587+56\sqrt{5}+30\sqrt{11}+17\sqrt{14}+9\sqrt{17}+3\sqrt{26}+6\sqrt{34}+3\sqrt{38}+12\sqrt{42}+6\sqrt{57}+2\sqrt{105}+3\sqrt{149}+\sqrt{157}+\ll 18\gg+\sqrt{2170}+\sqrt{2195}+\sqrt{2301}+\sqrt{3107}+\sqrt{3189}+\sqrt{5722}+\sqrt{7739}+\sqrt{8962}+\sqrt{10622}+\sqrt{12037}+\sqrt{17963}+\sqrt{25678}\right)$$

Out[17]= 1959.43244174754487728128

HullMeanWidth also produces exact output, but it is hard to simplify due to terms like (surd1)*ArcCos[(integer)/(surd2)]

```
In[18]:= HullMeanWidth[ps, ch3D] // Short
         N[%, 24]
```

Out[18]//Short=

$$\frac{1}{2}\left(\frac{3\,\text{ArcCos}\left[\frac{4}{5}\right]}{2\,\pi}+\frac{5\,\sqrt{13}\,\text{ArcCos}\left[\frac{6}{7}\right]}{2\,\pi}+\frac{\sqrt{17}\,\text{ArcCos}\left[\frac{8}{9}\right]}{\pi}+\ll 83\gg+\frac{3\,\sqrt{13}\,\text{ArcCos}\left[\frac{1981}{\sqrt{3924829}}\right]}{2\,\pi}+\frac{\sqrt{\frac{23}{2}}\,\text{ArcCos}\left[\frac{4017}{\sqrt{16987105}}\right]}{\pi}\right)$$

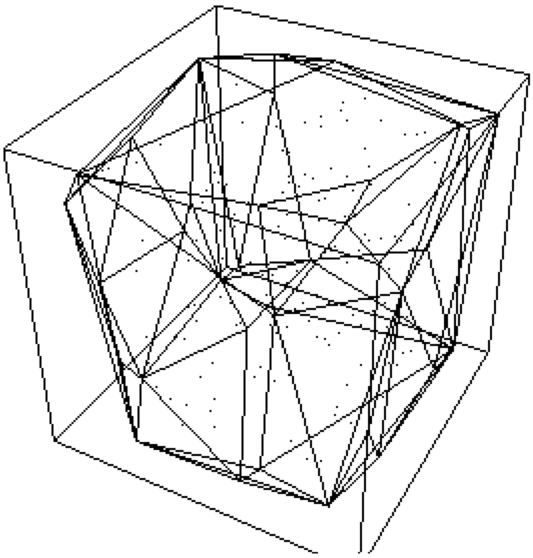
Out[19]= 26.4996605191315830162147

```
In[20]:= << RealTime3D`   (* <<Default3D` *)
```

```
In[21]:= << Graphics`Shapes`
```

rotating the next plot gives an intuitive validation of the result:

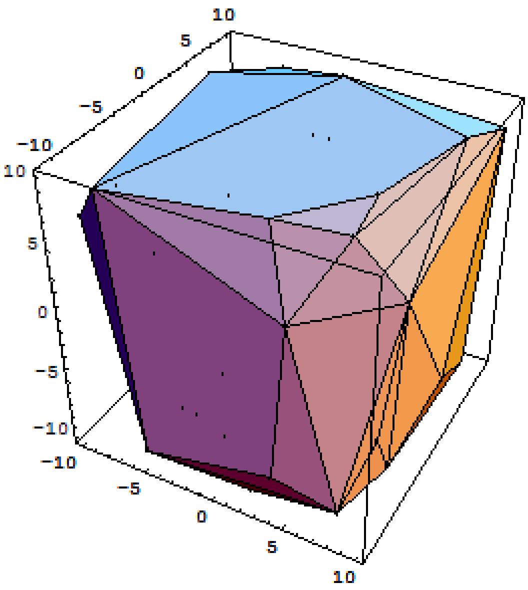
```
In[22]:= dynamic =
          Show[Graphics3D[
            {PointSize[0.02], Map[Point, ps, {1}]}],
           WireFrame@
            Graphics3D[Polygon /@ (ps[[#]] & /@ ch3D)]]
```



Out[22]= - Graphics3D -

```
In[23]:= (* <<RealTime3D`   *) << Default3D`
```

```
In[24]:= HullShow[ps, ch3D]
```



Out[24]= - Graphics3D -

list of points on the Hull but not spanning the Hull: inside a face or on an edge:

```
In[25]:= DiscreteMath`ConvexHull3D`Private`badones

Out[25]= {14, 18, 25, 27, 28, 47, 58, 74, 99, 117,
          140, 145, 164, 169, 172, 186, 191, 197}
```

five easy pieces ...

```
In[26]:= << Geometry`Polytopes`

In[27]:= norm[vec_] := Sqrt[vec.vec]
```

Platonic solids rescaled to radius = 1 (of circumscribed sphere)
http://en.wikipedia.org/wiki/Platonic_solid
http://mathworld.wolfram.com/PlatonicSolid.html

Zero'th easy piece :
the points ...

```
In[28]:= pts = {{1, 1, 1}, {-1, 1, 1}, {-1, -1, 1},
          {1, -1, 1}, {-1, -1, -1}, {1, -1, -1},
          {1, 1, -1}, {-1, 1, -1}};
```

*... obviously form a cube of edge =2, so volume= 2^3=8; area= 6*2^2=24; radius= Sqrt(1^2+1^2+1^2)=Sqrt(3) ; less obviously, the hull mean width = 3, since total edge length is 12*2=24, and*
*the angle between normals to joining faces is 2π /4, so 1/(4π) * (2π/4) * 24 = 24/8 = 3.*

```
In[29]:= ch3D = ConvexHull3D[pts]

Out[29]= {{1, 2, 3, 4}, {1, 4, 6, 7}, {1, 7, 8, 2},
          {2, 8, 5, 3}, {3, 5, 6, 4}, {5, 8, 7, 6}}

In[30]:= radius = Union[norm[{0, 0, 0} - #] & /@ pts]
```

Out[30]= $\{\sqrt{3}\}$

In[31]:= edgelength[{1, 2}, pts]

Out[31]= 2

In[32]:= {HullVolume[pts, ch3D], HullArea[pts, ch3D], HullMeanWidth[pts, ch3D]}

Out[32]= {8, 24, 3}

Platonic solids rescaled to radius=1 (of circumscribed sphere)

In[33]:= << Geometry`Polytopes`

The polyhedra functions Volume, InscribedRadius, and CircumscribedRadius return information for a polyhedron with edges of length 1. The list of coordinates returned by Vertices is conventional for the specified polyhedron and does not necessarily correspond to a polyhedron with unit edge length.

Tetrahedron

In[34]:= shape = Tetrahedron;

In[35]:= ps0 = Vertices[shape]

Out[35]= $\left\{\left\{0, 0, \sqrt{3}\right\}, \left\{0, 2\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}\right\}, \left\{-\sqrt{2}, -\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}\right\}, \left\{\sqrt{2}, -\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}\right\}\right\}$

In[36]:= {radius} = Union[Sqrt[({0, 0, 0} - #).({0, 0, 0} - #)] & /@ ps0]

Out[36]= $\left\{\sqrt{3}\right\}$

In[37]:= ps = ps0 / radius

Out[37]= $\left\{\{0, 0, 1\}, \left\{0, \frac{2\sqrt{2}}{3}, -\frac{1}{3}\right\}, \left\{-\sqrt{\frac{2}{3}}, -\frac{\sqrt{2}}{3}, -\frac{1}{3}\right\}, \left\{\sqrt{\frac{2}{3}}, -\frac{\sqrt{2}}{3}, -\frac{1}{3}\right\}\right\}$

In[38]:= ch3D = ConvexHull3D[ps]

Out[38]= {{1, 2, 3}, {1, 3, 4}, {1, 4, 2}, {2, 4, 3}}

In[39]:= Faces[shape]
Union[%] == Union[%%]

Out[39]= {{1, 2, 3}, {1, 3, 4}, {1, 4, 2}, {2, 4, 3}}

Out[40]= True

if both the above do not agree, then there is a problem with the circulation of Faces[shape] = sign of surface normal

In[41]:= {edge = edgelength[{1, 2}, ps] // FullSimplify, N[edge, 18]}

Out[41]= $\left\{2\sqrt{\frac{2}{3}}, 1.63299316185545207\right\}$

```
In[42]:= {it = HullArea[ps, ch3D], N[it, 18]}
```

Out[42]= $\left\{\frac{8}{\sqrt{3}}, 4.61880215351700612\right\}$

```
In[43]:= {it = HullVolume[ps, ch3D], N[it, 18]}
```

Out[43]= $\left\{\frac{8}{9\sqrt{3}}, 0.513200239279667346\right\}$

```
In[44]:= {it =
      {edge * CircumscribedRadius[#],
        NumberOfFaces[#] * edge^2 * Area[#],
        edge^3 * Volume[#]} & @shape ,
    N[it, 18]} // FullSimplify
```

Out[44]= $\left\{\left\{1, \frac{8}{\sqrt{3}}, \frac{8}{9\sqrt{3}}\right\}, \{1.00000000000000000, 4.61880215351700612, 0.513200239279667346\}\right\}$

```
FullSimplify[dihedralAngle[ps, ch3D, {1, 2}]]
```

Out[45]= ArcSec[-3]

```
In[46]:= {it = HullMeanWidth[ps, ch3D] // Simplify,
    N[it, 18]}
```

Out[46]= $\left\{\frac{\sqrt{6}\,\text{ArcCos}\left[-\frac{1}{3}\right]}{\pi}, 1.48971462263410635\right\}$

Octahedron

```
In[47]:= shape = Octahedron;
```

```
In[48]:= ps0 = Vertices[shape]
```

Out[48]= $\left\{\left\{0, 0, \sqrt{2}\right\}, \left\{\sqrt{2}, 0, 0\right\}, \left\{0, \sqrt{2}, 0\right\}, \left\{0, 0, -\sqrt{2}\right\}, \left\{-\sqrt{2}, 0, 0\right\}, \left\{0, -\sqrt{2}, 0\right\}\right\}$

```
In[49]:= {radius} =
    Union[
     Sqrt[({0, 0, 0} - #).({0, 0, 0} - #)] & /@ ps0]
```

Out[49]= $\left\{\sqrt{2}\right\}$

```
In[50]:= ps = ps0 / radius
```

```
Out[50]= {{0, 0, 1}, {1, 0, 0}, {0, 1, 0},
   {0, 0, -1}, {-1, 0, 0}, {0, -1, 0}}
```

```
In[51]:= ch3D = ConvexHull3D[ps]
```

```
Out[51]= {{1, 2, 3}, {1, 3, 5}, {1, 5, 6}, {1, 6, 2},
   {2, 4, 3}, {2, 6, 4}, {3, 4, 5}, {4, 6, 5}}
```

In[52]:= {edge = edgelength[{1, 2}, ps] // FullSimplify,
N[edge, 18]}

Out[52]= $\{\sqrt{2}, 1.41421356237309505\}$

In[53]:= {it = HullArea[ps, ch3D], N[it, 18]}

Out[53]= $\{4\sqrt{3}, 6.92820323027550917\}$

In[54]:= {it = HullVolume[ps, ch3D], N[it, 18]}

Out[54]= $\{\frac{4}{3}, 1.33333333333333333\}$

In[55]:= {it =
{CircumscribedRadius[#] * edge,
NumberOfFaces[#] * edge ^ 2 * Area[#],
Volume[#] * edge ^ 3} & @ shape ,
N[it, 18]} // FullSimplify

Out[55]= $\{\{1, 4\sqrt{3}, \frac{4}{3}\}, \{1.00000000000000000, 6.92820323027550917, 1.33333333333333333\}\}$

FullSimplify[dihedralAngle[ps, ch3D, {1, 2}]]

Out[56]= ArcSec[3]

In[57]:= {it = HullMeanWidth[ps, ch3D] // Simplify,
N[it, 18]}

Out[57]= $\{\frac{3\sqrt{2}\,\mathrm{ArcCos}[\frac{1}{3}]}{\pi}, 1.66237927193871596\}$

Hexahedron or Cube

In[58]:= shape = Hexahedron;

In[59]:= ps0 = Vertices[shape]

Out[59]= $\{\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\}, \{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\}, \{-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\}, \{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\}, \{-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}, \{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}, \{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}, \{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}\}$

In[60]:= {radius} =
Union[
Sqrt[({0, 0, 0} - #).({0, 0, 0} - #)] & /@ ps0]

Out[60]= $\{\sqrt{\frac{3}{2}}\}$

In[61]:= ps = ps0 / radius

Out[61]= $\left\{\left\{\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}, \left\{-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}, \left\{-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}, \left\{\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}, \left\{-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right\}, \left\{\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right\}, \left\{\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right\}, \left\{-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right\}\right\}$

```
In[62]:= ch3D = ConvexHull3D[ps]

Out[62]= {{1, 2, 3, 4}, {1, 4, 6, 7}, {1, 7, 8, 2},
          {2, 8, 5, 3}, {3, 5, 6, 4}, {5, 8, 7, 6}}

In[63]:= {edge = edgelength[{1, 2}, ps] // FullSimplify,
          N[edge, 18]}
```

Out[63]= $\left\{\frac{2}{\sqrt{3}}, 1.15470053837925153\right\}$

```
In[64]:= {it = HullArea[ps, ch3D], N[it, 18]}

Out[64]= {8, 8.00000000000000000}

In[65]:= {it = HullVolume[ps, ch3D], N[it, 18]}
```

Out[65]= $\left\{\frac{8}{3\sqrt{3}}, 1.53960071783900204\right\}$

```
In[66]:= {it =
            {CircumscribedRadius[#] * edge,
               NumberOfFaces[#] * edge^2 * Area[#],
               Volume[#] * edge^3} & @shape ,
           N[it, 18]} // FullSimplify
```

Out[66]= $\left\{\left\{1, 8, \frac{8}{3\sqrt{3}}\right\}, \{1.00000000000000000, 8.00000000000000000, 1.53960071783900204\}\right\}$

```
FullSimplify[dihedralAngle[ps, ch3D, {1, 2}]]
```

Out[67]= $\frac{\pi}{2}$

```
In[68]:= {it = HullMeanWidth[ps, ch3D] // Simplify,
          N[it, 18]}
```

Out[68]= $\left\{\sqrt{3}, 1.73205080756887729\right\}$

Icosahedron

```
In[69]:= shape = Icosahedron;

In[70]:= (ps0 = Vertices[shape]) // Short
```

Out[70]//Short=

$$\left\{\left\{0, 0, \sqrt{\frac{5}{2} - \frac{\sqrt{5}}{2}}\right\}, \left\{\sqrt{\frac{2}{5}\left(5 - \sqrt{5}\right)}, 0, \sqrt{\frac{1}{2} - \frac{1}{2\sqrt{5}}}\right\}, \left\{\sqrt{\frac{1}{5}\left(5 - 2\sqrt{5}\right)}, 1, \sqrt{\frac{1}{2} - \frac{1}{2\sqrt{5}}}\right\}, \ll 6 \gg, \left\{-\sqrt{\frac{1}{5}\left(5 - 2\sqrt{5}\right)}, -1, -\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{5}}}\right\}, \left\{\sqrt{\frac{1}{2} + \frac{1}{2\sqrt{5}}}, \frac{1}{2}\left(1 - \sqrt{5}\right), -\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{5}}}\right\}, \left\{0, 0, -\sqrt{\frac{5}{2} - \frac{\sqrt{5}}{2}}\right\}\right\}$$

In[71]:=
```
{radius} =
 Union[
  Sqrt[({0, 0, 0} - #).({0, 0, 0} - #)] & /@
   ps0 // FullSimplify]
```

Out[71]= $\left\{\sqrt{\frac{5}{2} - \frac{\sqrt{5}}{2}}\right\}$

In[72]:=
```
(ps = ps0 / radius // FullSimplify ) //
 Short
```

Out[72]//Short=

$$\left\{\{0, 0, 1\}, \left\{\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\right\}, \left\{\frac{1}{10}\left(5 - \sqrt{5}\right), \sqrt{\frac{1}{10}\left(5 + \sqrt{5}\right)}, \frac{1}{\sqrt{5}}\right\}, \ll 6 \gg, \left\{\frac{1}{10}\left(-5 + \sqrt{5}\right), -\sqrt{\frac{1}{10}\left(5 + \sqrt{5}\right)}, -\frac{1}{\sqrt{5}}\right\}, \left\{\frac{1}{10}\left(5 + \sqrt{5}\right), -\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{5}}}, -\frac{1}{\sqrt{5}}\right\}, \{0, 0, -1\}\right\}$$

In[73]:=
```
ch3D = ConvexHull3D[ps]
```

Out[73]=
```
{{1, 2, 3}, {1, 3, 4}, {1, 4, 5},
 {1, 5, 6}, {1, 6, 2}, {2, 6, 11}, {2, 7, 3},
 {2, 11, 7}, {3, 7, 8}, {3, 8, 4}, {4, 8, 9},
 {4, 9, 5}, {5, 9, 10}, {5, 10, 6},
 {6, 10, 11}, {7, 11, 12}, {7, 12, 8},
 {8, 12, 9}, {9, 12, 10}, {10, 12, 11}}
```

In[74]:=
```
{edge = edgelength[{1, 2}, ps] // FullSimplify,
 N[edge, 18]}
```

Out[74]= $\left\{\sqrt{2 - \frac{2}{\sqrt{5}}}, 1.05146222423826721\right\}$

In[75]:=
```
{it = HullArea[ps, ch3D] // FullSimplify,
  N[it, 18]}
```

Out[75]= $\left\{-2\sqrt{3}\left(-5+\sqrt{5}\right),\ 9.57454138327393916\right\}$

In[76]:=
```
{(it = HullVolume[ps, ch3D]) // FullSimplify,
  N[it, 18]}
```

Out[76]= $\left\{\frac{2}{3}\sqrt{2\left(5+\sqrt{5}\right)},\ 2.53615071012040953\right\}$

In[77]:=
```
{it =
   {CircumscribedRadius[#] * edge,
      NumberOfFaces[#] * edge^2 * Area[#],
      Volume[#] * edge^3} &@shape ,
   N[it, 18]} // FullSimplify
```

Out[77]= $\left\{\left\{1,\ -2\sqrt{3}\left(-5+\sqrt{5}\right),\ \frac{2}{3}\sqrt{2\left(5+\sqrt{5}\right)}\right\},\ \{1.00000000000000000,\ 9.57454138327393916,\ 2.53615071012040953\}\right\}$

```
FullSimplify[dihedralAngle[ps, ch3D, {1, 2}]]
```

Out[78]= $\text{ArcSec}\left[\frac{3}{\sqrt{5}}\right]$

In[79]:=
```
{Timing[TimeConstrained[
    FullSimplify[
     it = HullMeanWidth[ps, ch3D]], 120]],
  N[Simplify@it, 18]}
```

Out[79]= $\left\{\left\{0.468\ \text{Second},\ \frac{3\sqrt{\frac{5}{2}\left(5-\sqrt{5}\right)}\ \text{ArcSec}\left[\frac{3}{\sqrt{5}}\right]}{\pi}\right\},\ 1.83174861235073834\right\}$

Dodecahedron

In[80]:=
```
shape = Dodecahedron;
```

In[81]:=
```
(ps0 = Vertices[shape]) // Short
```

Out[81]//Short=

$\left\{\left\{\sqrt{\frac{1}{2}-\frac{1}{2\sqrt{5}}},\ \frac{1}{2}\left(3-\sqrt{5}\right),\ \sqrt{\frac{1}{2}+\frac{1}{2\sqrt{5}}}\right\},\right.$
$\left\{-\sqrt{\frac{5}{2}-\frac{11}{2\sqrt{5}}},\ \frac{1}{2}\left(-1+\sqrt{5}\right),\ \sqrt{\frac{1}{2}+\frac{1}{2\sqrt{5}}}\right\},$
«16»,
$\left\{\sqrt{\frac{5}{2}-\frac{11}{2\sqrt{5}}},\ \frac{1}{2}\left(1-\sqrt{5}\right),\ -\sqrt{\frac{1}{2}+\frac{1}{2\sqrt{5}}}\right\},$
$\left.\left\{2\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)},\ 0,\ -\sqrt{\frac{1}{2}+\frac{1}{2\sqrt{5}}}\right\}\right\}$

In[82]:=
```
{radius} =
  Union[
   Sqrt[({0, 0, 0} - #).({0, 0, 0} - #)] & /@
    ps0 // FullSimplify]
```

Out[82]= $\left\{\sqrt{\frac{3}{2}\left(3-\sqrt{5}\right)}\right\}$

In[83]:=
```
(ps = ps0 / radius // FullSimplify ) //
  Short
```

Out[83]//Short=

$$\left\{\left\{\sqrt{\frac{1}{30}\left(5+\sqrt{5}\right)},\ \sqrt{\frac{1}{6}\left(3-\sqrt{5}\right)},\ \sqrt{\frac{1}{3}+\frac{2}{3\sqrt{5}}}\right\},\right.$$
$$\left\{-\sqrt{\frac{1}{15}\left(5-2\sqrt{5}\right)},\ \frac{1}{\sqrt{3}},\ \sqrt{\frac{1}{3}+\frac{2}{3\sqrt{5}}}\right\},$$
$$\left\{-\sqrt{\frac{2}{15}\left(5-\sqrt{5}\right)},\ 0,\ \sqrt{\frac{1}{3}+\frac{2}{3\sqrt{5}}}\right\},\ \ll 15 \gg,$$
$$\left\{\sqrt{\frac{1}{15}\left(5-2\sqrt{5}\right)},\ -\frac{1}{\sqrt{3}},\ -\sqrt{\frac{1}{3}+\frac{2}{3\sqrt{5}}}\right\},$$
$$\left.\left\{\sqrt{\frac{2}{15}\left(5-\sqrt{5}\right)},\ 0,\ -\sqrt{\frac{1}{3}+\frac{2}{3\sqrt{5}}}\right\}\right\}$$

In[84]:=
```
ch3D = ConvexHull3D[ps]
```

Out[84]=
```
{{1, 2, 3, 4, 5}, {1, 5, 10, 15, 6},
 {1, 6, 11, 7, 2}, {2, 7, 12, 8, 3},
 {3, 8, 13, 9, 4}, {4, 9, 14, 10, 5},
 {6, 15, 20, 16, 11}, {7, 11, 16, 17, 12},
 {8, 12, 17, 18, 13}, {9, 13, 18, 19, 14},
 {10, 14, 19, 20, 15}, {16, 20, 19, 18, 17}}
```

In[85]:=
```
{edge = edgelength[{1, 2}, ps] // FullSimplify,
  N[edge, 18]}
```

Out[85]= $\left\{\sqrt{2-\frac{2\sqrt{5}}{3}},\ 0.713644179546179864\right\}$

In[86]:=
```
{it = HullArea[ps, ch3D] // FullSimplify,
  N[it, 18]}
```

Out[86]= $\left\{2\sqrt{50-10\sqrt{5}},\ 10.5146222423826721\right\}$

In[87]:=
```
{(it = HullVolume[ps, ch3D]) // FullSimplify,
  N[it, 18]}
```

Out[87]= $\left\{\frac{2}{3}\sqrt{\frac{10}{3}\left(3+\sqrt{5}\right)},\ 2.78516386312262297\right\}$

In[88]:=
```
{it =
   {CircumscribedRadius[#] * edge,
     NumberOfFaces[#] * edge^2 * Area[#],
     Volume[#] * edge^3} & @shape ,
  N[it, 18]} // FullSimplify
```

Out[88]= $\left\{\left\{1,\ 2\sqrt{50-10\sqrt{5}},\ \frac{2}{3}\sqrt{\frac{10}{3}\left(3+\sqrt{5}\right)}\right\},\right.$
{1.00000000000000000,
10.5146222423826721, 2.78516386312262297}}

```
FullSimplify[dihedralAngle[ps, ch3D, {1, 2}]]
```

Out[89]= $\mathrm{ArcSec}\left[\sqrt{5}\right]$

In[90]:=
```
{Timing[TimeConstrained[
   FullSimplify[
    it = HullMeanWidth[ps, ch3D]], 120]],
 N[Simplify@it, 18]}
```

Out[90]= $\left\{\left\{34.695\ \mathrm{Second},\ \frac{5\sqrt{\frac{3}{2}\left(3-\sqrt{5}\right)}\ \mathrm{ArcSec}\left[\sqrt{5}\right]}{\pi}\right\},\right.$
1.88624925030367027}

From the definition of Hull Mean Width = 1/(4*pi) multipled by the sum over all edges of
(length of an edge) * (corresponding angle between outward normals to the two faces that intersect along the edge)
we can verify (30*Sqrt[2 - (2*Sqrt[5])/3]*ArcSec[Sqrt[5]])/(4*Pi) = 1.88624925030367027

Converted by *Mathematica* November 7, 2012