inits

```
<< Combinatorica`

content[{}] := {{}};
content[(p_)?PartitionQ] :=
  Block[{le = Max[p], ferr = (PadLeft[1 + 0 * Range[#1], Max[p]] &) /@ p},
   DeleteCases[ MapIndexed[-le + Range[le, 1, -1] - #1 - Tr[#2] &, 0 * ferr] * ferr,
     0, -1] + le];

e[n_, v_] :=
 Tr[Times @@@ Select[Subsets[Table[Subscript[x, j], {j, v}]], Length[#] == n &]];
e[par_?PartitionQ, v_] := Times @@ (e[#, v] & /@par)
```

6/16/13 17:31:13 In[367]:=

```
f[ li_List, fun_, par_, k_ ] :=
 fun[par, k] /. Thread[Array[Subscript[x, #1] &, Length[li]] -> li]
```

main

$$t\,(t+1) \sum_{k=0}^{n} 1/n! \sum_{\lambda \vdash n} f(\lambda)^2\ F(k;\, t + c_u{}^2 : u \in \lambda) =$$

$$\sum_{k=0}^{n+2} (-1)^k/(n+2)! \sum_{\lambda \vdash n+2} f(\lambda)^2\ F(k;\, t + c_u{}^2 : u \in \lambda) =$$

$$t\,(t+1) \sum_{k=0}^{n+1} (-1)^{n+1+k}\, t^{k-1}\, S_1(n+1,\, k)$$

with F(k; *list-of-n-values*) is defined here as the Elementary Symmetric Function e(n-k, n) with its variables taking the values specified in "*list-of-n-values*",
and S_1 the Stirling numbers of the first kind, and f(λ) the number of Tableaux of shape λ, so $f(\lambda) = n! / \prod_{u \in \lambda} h_u$.

6/16/13 17:31:20 In[368]:=

```
Simplify[
 Sum[(-1)^(k) / 5!
    Tr[ NumberOfTableaux[#]^2 f[t + Flatten[content[#]]^2, e, 5 - k, 5 ] & /@
      Partitions[5] ] , {k, 0, 5}] ==
  t * (t - 1) * Sum[(-1)^(4 + m) t^(m - 1) StirlingS1[4, m], {m, 4}] ==
 Sum[(+1)^(k) / 3!
     Tr[ NumberOfTableaux[#]^2 f[t + Flatten[content[#]]^2, e, 3 - k, 3 ] & /@
       Partitions[3] ] , {k, 0, 3}]  * t * (t - 1) ]
```

6/16/13 17:31:20 Out[368]=

```
True
```

6/16/13 17:31:22 In[369]:=

```
n=5;
```

6/16/13 17:31:25 In[370]:=

```
t * (t - 1) * Sum[(+1)^(k) / n!
    Tr[ NumberOfTableaux[#]^2 f[t + Flatten[content[#]]^2, e, n - k, n ] & /@
      Partitions[ n ] ] , {k, 0, n}]   // Expand
```

6/16/13 17:31:25 Out[370]=

$-120\,t - 154\,t^2 + 49\,t^3 + 140\,t^4 + 70\,t^5 + 14\,t^6 + t^7$

6/16/13 17:31:27 In[371]:=

```
Sum[(-1)^(k) / (n+2)!
  Tr[ NumberOfTableaux[#]^2 f[t + Flatten[content[#]]^2, e, (n+2) - k, n+2 ] &/@
   Partitions[n+2] ] , {k, 0, n+2}] // Expand
```

6/16/13 17:31:27 Out[371]=

$-120\,t - 154\,t^2 + 49\,t^3 + 140\,t^4 + 70\,t^5 + 14\,t^6 + t^7$

6/16/13 17:31:30 In[372]:=

```
t * (t-1) * Sum[(-1)^(n+1+m) t^(m-1) StirlingS1[n+1, m], {m, n+1}]  // Expand
```

6/16/13 17:31:30 Out[372]=

$-120\,t - 154\,t^2 + 49\,t^3 + 140\,t^4 + 70\,t^5 + 14\,t^6 + t^7$

References

http://arxiv.org/pdf/0807.0383v3.pdf "Some Combinatorial Properties of Hook Lengths, Contents, and Parts of Partitions", Richard P. Stanley

http://www.emis.de/journals/SLC/wpapers/s61vortrag/han.pdf "A promenade in the garden of hook length formulas", Guo-Niu Han

play with it some

decomposition: replacing Sum with Table

values at t=2

values at t=3