A stable and consistently decisive collective decision procedure

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Abstract

A staged collective searching and decision-making procedure will be introduced that evaluates several different proposals simultaneously and refines them in subsequent steps. The procedure strives for a core-point and is also suitable for very complex situations. The procedure is always decisive. The preferred ranking order of decision-makers is a public given that offers more refined information within a democracy. Shadow votes are also possible in the same way, enabling candidate-decision-makers to make public their preference profile.

Keywords : mechanism design, implementation, core, n-person bargaining game, social choice function (SCF), democracy, decisiveness, stability

JEL Classification Numbers : C7, D78, D82

1 Introduction

A well-functioning democracy means an interplay between well-informed, participating citizens and well-functioning institutions. To these institutions, the majoritarian voting system is the workhorse of decision making. This method is ideal whenever the decision at hand is restricted by two mutually exclusive alternatives, such as: should cars drive on the right side of the road, or on the left side? May's theorem [May52] states that in such situations the majoritarian system is the only method that meets a number of reasonable conditions. Whenever there are more alternatives, this majoritarian system may be problematic. It cannot handle high degrees of complexity. Instability may also arise when there are multiple parties involved of which none have the majority. The quest for a majority may fail or take a long time and majorities may turn out not to be durable. An easy example is a case in which three parties A, B and C all have one-third of the seats. A and B may decide to form a majority, but there is no guarantee that this will be a durable situation and just a trace of mistrust or a small opportunity may cause other majorities to arise (BC, CA). A durable prevention of instability gives rise to the call for a strong leader, or may even justify a dictator. History shows us that this has in fact happened in the past. In a majoritarian system there is a constant urge to form large parties, but this goes hand in hand with a high degree of intraparty discipline and it comes at the expense of independence and a loss of diversity in elected representatives. The other extreme is a majority so large that it has two-thirds of the seats, which in many cases is enough to amend the constitution. This situation can easily be abused, for instance to change voting districts and voting rules in the face of "the next democratic elections", the substitution of key figures such as judges in a supreme court etc. In order to rule out these problems, in this article we will discuss a decision-making method in which several options can be considered at the same time. The parliamentarians (or decision-makers in any other context) will rank the various suggestions and the procedure (or social choice function) subsequently will yield a decision. A major difference from the majoritarian system is the fact that everyone gets to have a say in the matter at hand and the fact that the concept of opposition becomes meaningless. The working method will be described here with its practical use in mind. One of the theoretical difficulties in procedures with n players is the risk of manipulative use of the procedure by giving an input different from a person's own honest preferences. This has been described as being an inevitable risk long ago. The chance on, and the effect of, possible manipulation might not be so profound because the procedure includes various ballots and as such makes it fairly difficult for the manipulator to estimate the specific effect of the manipulation. Furthermore the effect of any such manipulation is even harder to predict by the potential manipulator on account of the unknown number of ballots in advance. Moreover, it is a public procedure. All proposals and preferred ranking orders will be made known to all participants as well as the public. This too discourages manipulation.

2 Organisation

A given number of N voters (for instance: parliamentarians) and N_P privileged voters (for instance: specially selected parliamentarians) who may formulate proposals :
$N_P <= N$ en $N_P >> 2$. Please notice "who may formulate proposals". This means that there is no obligation to do so, such as in very simplistic situations. In this text we will use symbols such as N_P and N both to indicate the set itself and its cardinal number. The context will make clear

which of the two is being meant. The bargaining set $\mathcal{P}$ is the union of all proposals that privileged voters may bring up. There are N_+ observers (the public) who cannot vote, but who can afterward elect their representatives. Formally they do not play a role in the procedure, but they can observe its course and they can also influence it (in the long run). This iterative procedure (or mechanism) will start with N_P (different) proposals brought forth by the privileged voters concerning some objective/subject (this may include a wide range of topics). The status quo solution p_0 is also always one of the alternatives. Therefore in the first ballot there will be a voting round for the proposals $\{p_0, p_1, \ldots, p_{N_{P-1}}, p_{N_P}\}$. There is a maximum number of ballots n. Each ballot therefore can lead to the addition of N_P new proposals. In the end of the procedure this leads to a maximum number of $n.N_P + 1$ proposals being assessed. After each ballot it will be decided whether there will be another ballot. This encompasses an additional, but also a simple vote in accordance with the q-rule, in which the number of votes is larger than, or equals, $q_i N$, in which $0 <= q_i <= 1$ at the end of ballot i ($i = 1 \ldots n-1$). Moreover, the qi form a monotonically increasing sequence in which we pose that $q_1 = 0$, which ultimately comes down to the fact that there is always a second ballot. An example of a q-vector for instance is $[0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}]$. In this case there are maximally 5 ballots, but for the final ballot to occur, $\frac{3}{4}$ of the voters have to agree at the end of the fourth ballot. The chance that there will be a next ballot with new proposals is therefore reduced in each subsequent phase. This urges the N_P participants (group leaders) to formulate increasingly better proposals that deviate from their ideal position but that ultimately have a better chance of making it through the final ballot. There is always a second ballot. After all, the first ballot is merely an introduction round in which all voters take notice of each other's proposals and the ideal positions of the privileged voters.

3 Ranking procedure

During the first ballot all voters will rank all proposals. In the subsequent ballots they will rearrange their ranking by adding the new proposals to it. Previous rankings therefore cannot be altered. It is however possible to assign proposals the same rank. Voter j therefore can have the following ranking at any given time:

$$(p_{j,1}...p_{j,g_1}) \succ (p_{j,g_1+1}...p_{j,g_1+g_2}) \succ (p_{j,g_1+g_2+1}...p_{j,g_1+g_2+g_3}) \succ \ldots \tag{1}$$

The social choice function (SCF) has to meet additional criteria. It has to be possible for each voter to differentiate between a large number of proposals. This excludes methods such as approval voting (only 0 or 1 points possible). In the Borda method ([Tid06] [MS97] [Saa09]) points are being assigned in accordance with a mathematical set (for instance 10, 9, ..., 2, 1 in case of

10 proposals). This linearly increasing sequence does however pose a few problems. Suppose p_k and p_l are next to each other, but in the next ballot p_m is positioned in between, then their relative weight difference changes from 1 to 2. Another problem may present itself close to an optimum with a parabolic peak. The utility of new proposals to the voter will increase less rapidly near such an optimum. Still the voter is confined to weight increments of 1, which does not adequately reflect his sense of additional utility of these proposals near the (parabolic) optimum. That's why the SCF is being used, which is only based on pair-wise comparisons. These pair-wise comparisons are being summarised in a table T. We assume that for each k and l, given $p_k \succ p_l$ the element $T_{k,l}$ in table T is being increased with 1. In case of equality $p_k = p_l$, both $T_{k,l}$ and $T_{l,k}$ will be raised by $\frac{1}{2}$. In case of convention we will take the diagonal elements $T_{k,k} = 0$. Suppose a voter ranks 4 options a, b, c and d as follows

$$b \succ (a = c) \succ d \tag{2}$$

Then table T_j for this voter j reads:

$$T_j = \begin{vmatrix} 0 & 0 & \frac{1}{2} & 1 \\ 1 & 0 & 1 & 1 \\ \frac{1}{2} & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix} \tag{3}$$

in which for instance the first sequence and the first column correspond to a etc. The first row of T should be read as follows: The first number 0 is unusable (a compared with itself), the second number indicates that a is being beat by b, the third number indicates that a and c are equivalent and the fourth number indicates that a is being chosen over d. With 100 voters, the table could look like this:

$$T = \sum_{j=1}^{j=100} T_j = \begin{vmatrix} 0 & 62 & 59\frac{1}{2} & 10 \\ 38 & 0 & 65 & 16 \\ 40\frac{1}{2} & 35 & 0 & 16 \\ 90 & 84 & 84 & 0 \end{vmatrix} \tag{4}$$

If in the previous example the proposals e, f, g and h are also added to a subsequent ballot, then the new ranking of this voter in question could be as follows:

$$g \succ \mathbf{b} \succ (\mathbf{a} = \mathbf{c} = h) \succ \mathbf{d} \succ (e = f) \tag{5}$$

The existing ranking of a, b, c and d will remain intact.

4 Social Choice Function

We now repeatedly use the same social choice function (SCF) to come to a decision as to which proposal to choose. We may choose a method such as Kemeny-Young [Kem59] [You77]. Because we're looking for maximum stability under varying circumstances and because not the complete ranking but rather the eventual stable solution is the ultimate goal, we will shorty revisit another family of methods. We will look for proposals that cause the least resistance. We will use the columns of table T. A first indicator for resistance against a certain proposal is the total sum of all values within a column. The winning proposal is whichever one has the lowest total sum. We also envision an additional rule that makes sure there is always only one winner in case of two proposals having the same total count.

$$\min_j \sum_{k=1}^{k=n.N_P+1} p_{j,k} \tag{6}$$

This case is similar to the Copeland method ([Tid06] p. 206-209) ([MS97]). In the previous example the fourth column (option d) yields the lowest total count, being 42. The other extreme is to count the largest element from each column and to choose the solution from whatever column has the lowest maximum.

$$\min_j \max_k p_{j,k} \tag{7}$$

In the example the fourth column is once again chosen as being the best solution because the largest element is 16 and smaller than the largest elements of the other columns. This method is similar to the Maximin-method ([Tid06] p. 212-213) and in this case searching for a stable solution is equivalent to looking for a core-point for a q-rule. One might understand this as follows: a q-rule is a (super) majoritarian voting system in which at least a certain fraction q of the voters has to vote in favour of a certain proposal. A core for a q-rule is an element x from the set of all possible options for which there is no other element that beats x with a q-majority. We can now create a table T of this set with all pair-wise comparisons that yield a victory and we would find that there is no single values in column x that is large enough to form a q-majority. Please note that we allow draws to be counted as $\frac{1}{2}$ and that there thus is a slight deviation from this definition. Between these two extremes there are also other options. We observe all values in a column sorted from highest to lowest and take as a benchmark the sum of the f% highest values (rounded to the next whole number). If we sort each column and only use the half with the highest values, once again the fourth column (letter d) turns out to be the winner, with a total count of 32.

$$\min_j \sum_{k=f\%}^{k=100\%} p_{j,k} \tag{8}$$

This hybrid method is a compromise between maximal efficiency and robustness. Note that the majoritarian system is a special case and still is an option. Alternative p_0 equals the status quo or a no-vote and p_1 equals a new alternative. Table T for 100 voters could look as follows:

$$T = \begin{vmatrix} 0 & 35\frac{1}{2} \\ 64\frac{1}{2} & 0 \end{vmatrix} \tag{9}$$

In this case the Copeland method, the maximin-method and the hybrid methods in between all yield the same result.

5 Dynamics

In the case of this procedure we may envision two extreme situations. We will assume that all participants can individually rank all possible proposals (with possibly ex-aequos). The first extreme situation is the case in which everyone knows all preference rankings from everyone else. In that case each participant can know the solution and, assuming that everyone hands in a ranking that truly reflects his or her own preferences, all iterations instantly become redundant as soon as one participant proposes the ultimate best solution. Assuming that there is only one ultimate solution. The other extreme situation is the case in which participants have absolutely no information at all concerning the preferences of the other participants. Take for instance the situation in which each participant has ranked a series of codes consisting of letters and numbers: $8Z9C \succ BRJ \succ WXH5 \succ ....$ In this case the only rational way of working for each participant is to bring up his first choice in the first round, his second-best choice in the second round etc. After each ballot our method yields a total ranking of the proposals with the accompanying maximum element of this ranking. Indeed, it is possible to reiterate the social choice function (SCF) and in this way realise a social choice correspondence (SCC). Namely by first determining a first element using the complete table T and by subsequently applying this procedure to T with the column and row of the winner of the previous iteration deleted. Note that a voter's initial ranking of the proposals isn't allowed to be changed during the procedure. However, the collective ranking reached through the repeated application of the SCF may differ from rankings from previous steps (although it often will not, or will only to a small degree). This can be avoided by changing the rules for setting up an SCC, so that the old ranking of the previous step as well as the new ranking of the newly added elements remain in the merged order [Mas99]. The merging into the old order will take place from the largest of the new elements to the smallest. Whenever there are two or more ways to merge, each new element will be added into the ranking as far upfront as possible. This ranking will be expanded with each new ballot and the maximum element will either remain in the first place or it will be replaced by a new maximum element. An (extreme) example: First ballot with the ranking to be preserved:

$$a \succ b \succ c \succ d \tag{10}$$

Second ballot:

$$e \succ d \succ f \succ b \succ g \succ c \succ h \succ a \tag{11}$$

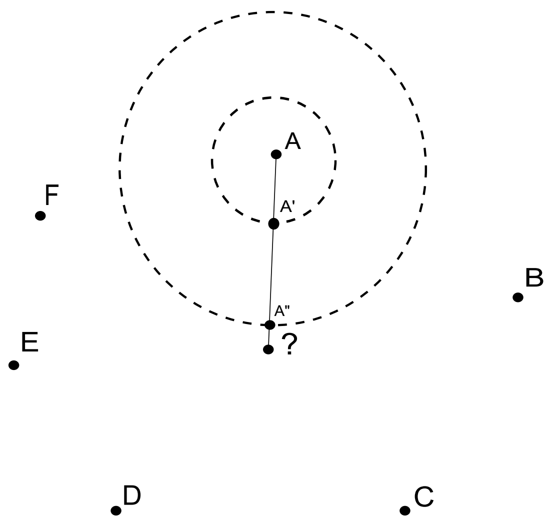
Additional ranking to be preserved:

$$e \succ f \succ g \succ h \tag{12}$$

e will certainly be the first. d will be followed by f, but because the order of the first ballot has to be respected, f will also be behind a, b and c. Final order:

$$e \succ a \succ b \succ c \succ d \succ f \succ g \succ h \tag{13}$$

For a large bargaining set $Card(\mathcal{P}) > n.N_P$ the chance of improvement is almost certainly positive with each ballot. The situation in which each participant does have some degree of information on the preferences of the others is more realistic. As an example we will take a two-dimensional situation with 6 participants A, B, C, D and F who are in the indicated positions (see figure).



Their utility functions diminish with Euclidian distance from their relative positions. The idea is to determine the optimal point that can be reached with this SCF using the method described here. Player A suspects they will eventually end up at **?** However, he reluctantly leaves his ideal position **A**. To him, all positions on a circle with centre **A** are equally good. But within the smallest circle he best chooses point **A'** because this is situated closest to the suspected balance point **?**. Of course he could opt to venture further from his ideal point and choose point **A"** on the larger circle. Each participant has more risky and less risky alternatives. The participants are at risk of their proposals not being preserved and that there will be no next ballot. By choosing a more realistic point **A"**, a participant improves his own chances. Participant A could choose point **?** right from the start, but he would have to keep in mind that he might also misjudge the situation and thus agree with a solution that is too favourable for the other participants. In a procedure with humans we should restrict ourselves to a finite number of steps. The maximum number of steps should at least be sufficient to bring most decisions to a good end. Assuming that the utility function of each voter is limited (bounded utility) and thus that the last steps will usually yield less improvement (diminishing returns), we have reason to be optimistic. The increasing q-values per step avoid procrastination, they force each privileged voter to come up with solutions close to the suspected optimum as soon as possible. Setting the number of steps and the q-values right can only be achieved through acquiring practical experience. For constitutional amendments the maximum number could be increased and an SCF could be selected that more closely resembles the maximin-method.

6 Complexity

Here we will look at complexity in the same way as the article by Saari [Saa97]. In this article too $N_P = N$, and stability of the core will be assessed for a q-rule. The maximin-method (above) is an implementation to find such a core. However, using this method it is impossible to know in advance which "q-value" can be reached in the best case. We assume that all proposals can be formulated in a K-dimensional space. Moreover we assume that each participant has an ideal point in this space as well as a monotonously constantly decreasing utility function that relates to the monotonously increasing distance along a line from the ideal point. Saari's article discusses conditions for stability of the core, namely whether a core can continue to exist of ideal points of utility functions change. For stability it is necessary that $K \leq (2q-1)N_P$ [Saa97]. Essentially this means that in case of increasing complexity K, the number of voters N_P has to increase proportionally. The procedure is a collective searching method in a K-dimensional space. If the N_P voters wish to refine the process, they could use the method by Nelder and Mead [NM65]. This method requires availability of a simplex with K + 1 points and function values with every iteration. When viewed in this context, K + 1 different viewpoints are also a minimum requirement in order to be able to make progress in a K-dimensional space. Nelder and Mead's method assumes that function values of the function to be optimised are available. In case of large complexity (large value for K), the thorough studying and ranking of proposals will become increasingly difficult. In that case it is possible to offer each voter a minimal set of proposals that he/she is obliged to rank. In addition it is still allowed to rank other proposals. The ranking of the obligatory proposals will take place using an incomplete balanced block design (IBBD). Such a block for instance contains c proposals. An example of Chochran and Cox [CC57] with a total of 9 proposals, 12 voters who rank 3 proposals each:

(1 2 3)	(4 5 6)	(7 8 9)
(1 4 7)	(2 5 8)	(3 6 9)
(1 5 9)	(7 2 6)	(4 8 3)
(1 8 6)	(4 2 9)	(7 5 3)

Each voter is assigned a block at random. Therefore it is possible that block 1 is assigned to voter 5. This voter may find proposal 5 excellent and nothing keeps him from also ranking proposal 5 in addition to the three obligatory proposals (1,2,3). In general this will mean that there are N blocks (the same as the number of voters). The following relationship exists for each IBBD: $c.N = r.N_P$, in which r indicates replicates (the number of times a proposal has to be assessed by a voter). This means that a proposal p_i is included in r blocks. In order to get to a "balanced" design, all proposals p_i would have to come past in combination with all of the other proposals p_j (i.e. all pairs) an equal amount of times. This is important because this procedure is based on pair-wise comparisons. There is one condition

$$\lambda = \frac{r(c-1)}{N_P - 1} \tag{14}$$

In the above example $\lambda = 1$. Imagine that in a complex situation each step includes 100 proposals (($N_P = 100$)) and that we want each proposal to be viewed at least 100 times (r = 100) and that each voter can process a maximum of 25 proposals (c = 25). From $N = \frac{N_P.r}{c}$ it follows that $N \geq \frac{100.100}{25} = 400$. In this case it holds true that $\lambda = \frac{100.24}{99} = 24.24$.
Suppose we want to raise the value of λ, then we have no choice than to also raise r and therefore also N. N will become 1650. It is clear that we can process any degree of high complexity (in the previous sense) as long as we can increase the number of voters. This is in stark contrast to the go-to way of dealing with complex situations in classical democracies, which is instituting an (all) powerful president or Führer. It is impossible to find an exact IBBD for all random figures. But it is always possible to set up a block design that does approximate the ideal situation well. One of the results is that the table T will include inequations. The inequations will be solved by simultaneously and repeatedly adding a $\frac{1}{2}$ to $T_{i1,j1}$ and $T_{j1,i1}$ until $T_{i1,j1} + T_{j1,i1} = N$.

7 Incomplete ranking

What if a voter chooses not to participate in a ballot? In that case all pairs that have not been compared will be assigned $\frac{1}{2}$. The same will happen when an IBBD is being used. A T-table like that isn't necessarily consistent anymore with a certain order. If we review this example:

$$g \succ \mathbf{b} \succ (\mathbf{a} = \mathbf{c} = h) \succ \mathbf{d} \succ (e = f) \tag{15}$$

Then the pair-wise table is

$$T = \begin{vmatrix} 0 & 0 & \frac{1}{2} & 1 & 1 & 1 & 0 & \frac{1}{2} \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ \frac{1}{2} & 0 & 0 & 1 & 1 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} & 1 & 1 & 1 & 0 & 0 \end{vmatrix} \tag{16}$$

In case of non-participation in the first ballot, this table will be:

$$T = \begin{vmatrix} 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 1 & 0 & 0 \end{vmatrix} \tag{17}$$

At that moment the individual ranking is lost. This is an impossible ranking because a=g and a=h but the same can happen when using IBBDs. Also note that whenever an SCF is being used to also record a ranking among the various proposals, that this order isn't necessarily retained in the next ballot.

8 Weighted voting : the council of EU ministers

The Council of the European Union currently makes decisions using the qualified majority voting method. It is possible to adjust the method of pair-wise comparisons to the relative weight of each country. In that case the numbers in each T_j table (j indicates the country) will be multiplied with, for instance, the number of inhabitants of that respective country. Some of the 28 countries, sorted from small to large in millions of inhabitants on 1 January 2016 (source Eurostat).

Germany	82,175,684
France	66,759,950
UnitedKingdom	65,382,556
Italy	60,665,551
...	...
Estonia	1,315,944
Cyprus	848,319
Luxembourg	576,249
Malta	434,403

The pair-wise table T would be (in millions):

$$T = 82.2\ T_{Ger}\ +\ 66.8\ T_{Fra}\ +\ \dots\ +\ 0.6\ T_{Lux}\ +\ 0.4\ T_{Mal} \tag{18}$$

9 Independent democracies

9.1 English

Assume there are two independent democracies that pass through the same aforementioned voting ballots simultaneously. To what degree can they come to common solutions? Democracy A works with a growing number of proposals P_A throughout the procedure, and for B this is P_B. However, after each ballot, the voters of both parliaments create rankings from the set $P_A \bigcup P_B$. Assume that the best solution from the "own set" for A is $p_{A,opt}$ and for B is $p_{B,opt}$. Now consider the sets $P_{A,opt+}$ and $P_{B,opt+}$ which respectively consist of the solution $p_{A,opt}$ and all solutions that are ranked higher in the case of A and the same for the set $P_{B,opt+}$ for B. If $P_{A,opt+} \bigcap P_{B,opt+} \neq \varnothing$, then there is a better common solution available. In order to find this common solution, one may merge these two pair-wise tables T_A and T_B with a weight factor previously agreed on per table (e.g. taking into account the number of inhabitants). This method also teaches us that two independent democracies may in this way sometimes implement a common proposal, and may sometimes not. However in this way they miss out on certain opportunities, potentially $P_{A,opt+} \neq \varnothing$ of $P_{B,opt+} \neq \varnothing$ each time. Merging of A and B may be beneficial if A and B are certain that deciding together on average is beneficial for both parties. After a while this method may provide insight into this mechanism and both parties may agree to adopt this system.

9.2 Example

Suppose Democracy A with own propositions a, b and c and democracy Z with own propositions x, y and z. A orders all the propositions as

$$a \succ x \succ b \succ y \succ c \succ z \tag{19}$$

and Z comes out the order

$$x \succ a \succ y \succ b \succ z \succ c \tag{20}$$

then there is no common solution. $P_{A,opt+} = \{a\}$ and $P_{Z,opt+} = \{x\}$ and $P_{A,opt+} \bigcap P_{Z,opt+} = \varnothing$ When A and Z vote together and can add propositions, say proposition m then a possible result might be :

$$m \succ x \succ a \succ y \succ b \succ z \succ c \tag{21}$$

which is an improvement for both.

10 Possible consequences and additional possibilities

10.1 Observability and transparance

A nice and unique result of this method is the fact that the public can now take notice of the preferred rankings of their elected representatives. Voters can now assess which elected representative within a party fits their personal profile best.

10.2 Shadow-parliament

A new possibility is a shadow parliament in which future candidates can take part in the same parliamentarian votes. There they can present themselves and the public may use this information in the following elections.

10.3 Smaller parties

In a majoritarian system all parties want to become as large as they possibly can in order to become "incontournable" and preferably to get the absolute majority. With the procedure proposed here this will be less relevant. After all, everyone and each party now plays a role in the final decision-making procedure. Setting a minimal scale might be beneficial but it will rather be a matter of associations of politically active candidates. A minimum size requirement will still more readily gain the public's trust rather than the isolated actions of individuals. And a certain scale will also facilitate a better organisation (such as a common secretariat, etc ...).

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