

The Relation between the Amplitude and the Period of Solar Cycles *

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Abstract The maximum amplitudes of solar activity cycles are found to be well anti-correlated ($r = -0.72$) with the newly defined solar cycle lengths three cycles before (at lag -3) in 13-month running mean sunspot numbers during the past 190 years. This result could be used for predicting the maximum sunspot numbers. The amplitudes of Cycles 24 and 25 are estimated to be 149.5 ± 27.6 and 144.3 ± 27.6 , respectively.

Key words: Sun: activity — Sun: sunspots — Sun: general

1 INTRODUCTION

Predictions of solar activity level are crucial to low-Earth orbiting satellites, electric power transmission, radio communications, and satellite planning, etc. (Hathaway et al. 1999; Kane 2001; Li & Gu 1999). There are dozens of methods (Yule 1927; Gnevyshev & Ohl 1948; Ohl 1966; Ohl & Ohl 1979; Waldmeier 1939; Kane 1978, 1989; Feynman 1982; Thompson 1993; Wilson 1990; Lantos & Richard 1998; Shastri 1998; Hathaway et al. 1994, 1999; Li et al. 2000, 2001a, 2002b; Zhang & Wang 1999) of predicting the behavior of sunspot number (SN, hereafter), which can be divided into two categories: statistical methods and precursor methods. The latter performed more accurately than the former in Cycles 20–22; however, not so for Cycle 23 (Li et al. 2001b; Kane 2001).

Two important parameters describing the solar activity cycle are the maximum amplitude (R_m) and the solar cycle length (period between two successive minima, P hereafter). They are not quite independent, e.g., stronger cycles tend to be shorter and vice versa (Waldmeier 1939; Dicke 1978; Friis-Christensen & Lassen 1991; Solanki et al. 2002; Hathaway et al. 2002). Waldmeier (1939) noted that large amplitudes tend to take less time (ascending duration) to reach their maxima than do small amplitude ones. This phenomenon is called the “Waldmeier effect”. It is obvious that neither parameter is able to predict the other before the maximum has reached. The “Waldmeier effect” can not provide a good indication of R_m even though the correlation coefficient between R_m and the ascending phase duration is not small (Wilson 1992; Wang et al. 1975; Wang 1992).

To predict some parameters in future solar cycles, we have to start with some known parameters. A relationship between R_m and P at lag -1 (amplitude-period effect) states that the current cycle’s R_m is inversely proportional to the P of the previous cycle (Hathaway et al. 1994). With this relationship, the previous P can be used as a predictor of the following R_m . The “three-cycle quasi-periodicity”, found recently in solar and geophysical data (Ahluwalia 1998, 2000; Silverman 1992; Orfila et al. 2002; Solanki

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et al. 2002; Du 2006b; Du & Du 2006) though argued by Wilson & Hathway (1999), can also be taken as a way of predicting future R_m . A “five-cycle quasi-periodicity” in SN was suggested recently (Du 2006c). The “Gleissberg cycle” (the long term trend cycle of about 80-yr) can be used to find the secular trend of R_m (Gleissberg 1971).

The “Gleissberg cycle”, ranging in 60–120 yr (Usoskin & Mursula 2003), was proposed (Wolf 1862) and confirmed (Gleissberg 1939; 1940). It is also found in solar irradiance (Reid 1987) and in auroral records (Siscoe 1980; Feynman & Fougere 1984). It is even found to be related to the Earth’s temperature (Reid 1987; Friis-Christensen & Lassen 1991).

Other long trend cycles, longer than 100-yr, have been found both in ^{14}C (Feynman & Gabriel 1990), and in sunspot records (Bracewell & William 1986; Bracewell 1986) with such different methods as power spectral analysis (Cole 1973), MESA (Cohen & Lintz 1974), Fourier and wavelet formalisms (Ogurtsov et al. 2002). The 175–180 yr cycles are usually considered as fundamental (Wood 1972; Cohen & Lintz 1974; Feynman & Gabriel 1990) and are related to Sun’s oscillation about the center of mass of the solar system (Dicke 1978; Landscheidt 1999). The 314-yr periodicity, or the Elatina cycle found in ancient varve (William & Sonett 1985; Bracewell & William 1986; Bracewell 1986), was not believed to be a dominant factor in contemporary sunspot cycles (Wilson 1988).

The “Even-Odd” pair of Cycles 22–23 violated the “G-O rule” which states that the size of odd-numbered cycle is larger than that of the previous even-numbered cycle (Gnevyshev & Ohl 1948). This suggests that solar activities are modulated by secular trends (De Meyer 1998; Kane 2001; Komitov & Bonev 2001; Duhau 2003), which can be analyzed with linear fit (Wilson 1988; 1992; Hathaway et al. 1999) or trigonometric series modulation (Hathaway et al. 1999; De Meyer 1998) to improve predictions.

In order to find an effective predictor, we have re-examined the cross-correlation between R_m and P with the SN data. The relationship between R_m and P is analyzed in Section 2. The values of R_m in cycles 24–25 are also estimated in the same section. Final discussion and conclusions are presented in Section 3.

2 CROSS-CORRELATION BETWEEN AMPLITUDE AND PERIOD

Due to the fact that early international (Wolf) sunspot numbers are thought to be overestimated, we shall use only the “good” and “most reliable” data (Eddy 1976; Kane 1999) from Cycle 7 on, i.e., from 1818 to the present (2004). The monthly values are taken from <http://www.ngdc.noaa.gov/stp/SOLAR/getdata.html>, and smoothed over 13 months. There are a total 17 cycles including the ongoing Cycle 23.

The weighted epoch, E_{\min} , of a minimum of SN size R_0 , is defined as

$$E_{\min} = \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i E_i, \quad (1)$$

where $w_i = 1/(R_i - R_0)$ is the weight of epoch E_i with SN size R_i . The summations are over values R_i that satisfy the condition, $R_i - R_0 \leq 0.1 \max(R_i - R_0)$. When $R_i = R_0$, w_i is taken three times the maximum of the other weights: $w_i = 3w' = 3/(R' - R_0)$, R' being the SN size closest to R_0 (Du et al. 2006). With this definition, the period in cycle i , $P(i)$, is defined as the epoch difference between two successive minima, $E_{\min}(i)$ and $E_{\min}(i + 1)$:

$$P(i) = E_{\min}(i + 1) - E_{\min}(i). \quad (2)$$

The values of P and R_m are listed in Table 1. P of cycle 6 (152) is for reference only, and is not used in the model of this paper. The varying trend of R_m is indicated by the symbol in bracket (“+”, “−”). Positive sign indicates that current value is larger than the previous one, and minus sign indicates the opposite. Other quantities in Table 1 will be discussed later.

For predicting R_m , we have to use known values of P in some past cycles. The series of data must be long enough to extract the regression equation at high significance. The correlation coefficients between R_m and P with data from cycle 7 onward are plotted against lag number $L = -1, -2, \dots, -7$ in Figure 1. It clearly shows that a significant correlation coefficient, $r = -0.72$, exists at the lag $L = -3$ with confidence level greater than 99%: in other words, R_m correlates most closely with the P three cycles earlier.

Table 1 Values of P , R_m , R_F and δ_F

Cycle (1)	P (month) (2)	R_m (3)	R_F (4)	δ_F (5)
6	(152) ^a			
7	127	73		
8	117	145.2(+)		
9	150	132.3(-)	(65.6) ^b	(-66.7) ^b
10	134	98.4(-)	130.7(+)	32.3
11	139	141.3(+)	156.8(+)	15.5
12	126	73.8(-)	70.8(-)	-3.0
13	150	88.6(+)	112.5(+)	23.8
14	141	65.3(-)	99.4(-)	34.1
15	120	103.3(+)	133.3(+)	30.0
16	122	77.4(-)	70.8(-)	-6.6
17	126	118.3(+)	94.2(+)	-24.0
18	121	150.1(+)	148.9(+)	-1.1
19	126	202.6(+)	143.7(-)	-58.9
20	139	111.6(-)	133.3(-)	21.7
21	120	163.6(+)	146.3(+)	-17.3
22	122	159.4(-)	133.3(-)	-26.1
23		119.8(-)	99.4(-)	-20.4

^a This value is used only for extrapolation rather than in model.
^b Extrapolation from “a”, Eqs. (3) and (4).

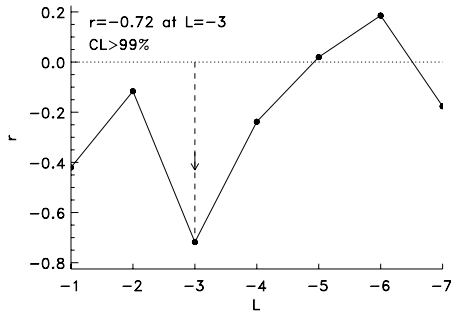


Fig. 1 Correlation coefficients between R_m and P at lag $L = -1, -2, \dots, -7$. Arrow marks the numerically largest r ($r = -0.72$, at a confidence level of 99%) at lag $L = -3$.

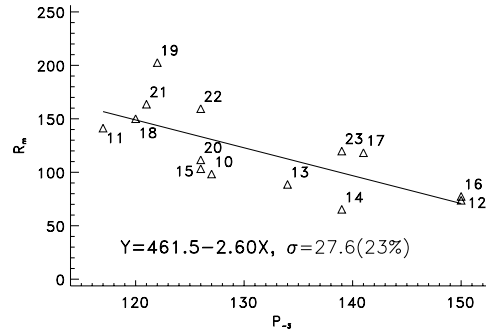


Fig. 2 Scatter plot of R_m vs. P_{-3} (triangles). The linear regression equation is $R_m = 461.5 - 2.60P_{-3}$, and the standard deviation is $\sigma = 27.6(23\%)$.

Figure 2 is a scatter plot of R_m vs. P_{-3} (from Cycle 7 on). The best linear fit is

$$R_F = 461.5 - 2.60P_{-3}, \tag{3}$$

where P_{-3} means the value of P three cycles earlier. The standard deviation is $\sigma = 27.6(23\%)$, here the percent in bracket is the relative error.

Substituting the values of P of Cycles 21 and 22 (120 and 122) into Equation (3), we can estimate the amplitudes of Cycles 24 and 25 to be 149.5 and 144.3, respectively. We note that the estimated amplitude, 149.5, of cycle 24 is close to the value 150 by Du (2006a), and larger than those by some authors, e.g., 87 by Duhau (2003), 101 by Wang et al. (2002), 105 by Kane (1999), 116 by Du (2006c), 132 by De Meyer (1998), and 139 by Solanki et al. (2002). Our predictions suggest that Cycle 24 should be stronger than Cycle 23, and that the “Even-Odd” pair of cycles 24–25 would violate the “G-O” rule as predicted by Duhau (2003) and Du (2006a).

In Figure 3, the abscissa represents the cycle number, the dashed line joins the fitted values R_F (Eq. (3), Column 4 in Table 1) and the solid line joins the observed values R_m . The fitted value in Cycle 9 is extrapolated from Equation (3) with the P of Cycle 6. The residuals,

$$\delta_F = R_F - R_m, \quad (4)$$

are listed in Column 5 of Table 1 and plotted against the cycle number in Figure 4.

It can be seen from Figures 1–4 that R_m is well correlated with P_{-3} (correlation coefficient -0.72). We shall call this fact the “three-cycle effect”. It is different from the generally adopted “Amplitude-period effect” (Hathaway et al. 1994) at lag -1 . The largest residual, -58.9 in Cycle 19, is caused by the unusually large R_m (202.6).

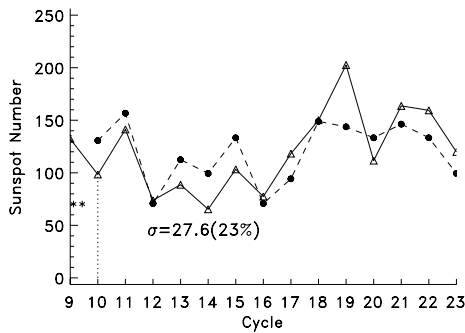


Fig. 3 Fitted values (R_F , filled circles) and observed ones (R_m , triangles) against cycle number. The standard deviation is 27.6(23%). R_F in Cycle 9 (“**”) is derived from Eq. (3) with P of Cycle 6.

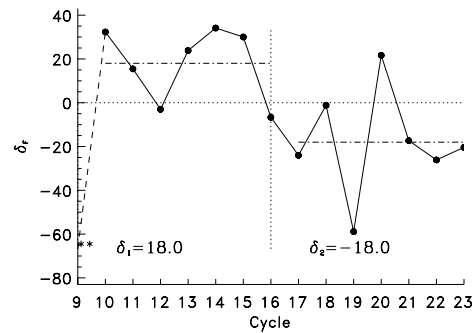


Fig. 4 The residuals, $\delta_F = R_F - R_m$, plotted against the cycle number. Also marked are the mean residual from Cycle 10 to 16 ($\delta_1 = 18.0$) and the mean residual from Cycle 17 to 23 ($\delta_2 = -18.0$)

Table 1 shows that, in 12 out of 14 cases, the varying trends (the plus and minus sign in Table 1) are the same in R_F and R_m . The two exceptions are the “Odd-Even” pair of cycles 9–10 which is caused by the long period of Cycle 6 ($p = 152$), and the “Even-Odd” pair of cycles 18–19 which is caused by the unusually large size of cycle 19 ($R_m = 202.6$). Thus, as judged by the varying trends, the “three-cycle effect” is well behaved.

The earlier cycles (Cycles 10 to 16) behave differently from the later cycles (Cycles 17 to 23): in the former, the fitted amplitudes are nearly all greater than the observed amplitudes (ref. Column 5 in Table 1 and Fig. 4), whereas in the latter, the fitted amplitudes are nearly all smaller than the observed amplitudes. The sign of δ_F reversed at Cycles 9 and 16. If the interval between these reversal points ($=7$ cycles) remains to hold in future, the next reversal point would be in the current cycle (23) but the residual, -20.4 , is not very small.

The mean residuals in the first 7-cycle-interval from Cycle 10 to 16, and in the second 7-cycle-interval from Cycle 17 to 23 are, respectively,

$$\delta_1 = \frac{1}{7} \sum_{i=10}^{16} \delta_F(i) = 18.0, \quad \delta_2 = \frac{1}{7} \sum_{i=17}^{23} \delta_F(i) = -18.0. \quad (5)$$

The absolute values of δ_1 and δ_2 are equal by chance. The two mean residuals (Eq. (5)) suggest a sign reversion of a cyclical drift about 18.0 every other seven cycles. This phenomenon seems to imply that the amplitude is modulated by a long trend cycle of double 7-cycles or 14-cycles (about 154 yr).

3 DISCUSSION AND CONCLUSIONS

We re-examined the relationship between maximum amplitudes and our recently defined periods (Du et al. 2006). The correlation coefficient ($R = -0.72$) between the amplitude and period is found to be significant at lag -3 , its size is similar to that obtained by Solanki et al. (higher than -0.63) (2002). This “three-cycle effect” (the associated period precede the amplitude by three cycles) is similarly found in the relationship between R_m and the descending time (Du & Du 2006). It is also found in the relationship between descending time and ascending time (Du 2006b). It implies that the solar dynamo indeed carries some memories from one cycle over into the next (De Meyer 1998; Hathaway et al. 1999; Solanki et al. 2002; Gleissberg 1971; Kane 2001; Duhau 2003).

Predictions of SN before Cycle 23 with statistical methods tend to be lower than observed ones, and predictions on Cycle 23 with precursor methods tend to be higher (Li et al. 2001b; Kane 2001; Wang et al. 2002). It is usually believed that sunspot numbers are modulated by long trend cycles (Oliver et al. 1998; De Meyer 1998; Hathaway et al. 1999; Kane 2001; Komitov & Bonev 2001; Duhau 2003). These cycles have a large range from 60-yr (Usoskin & Mursula 2003) to 350-yr (Feynman & Gabriel 1990). Our results show that the fitted amplitudes have a cyclical drift (± 18.0) every other seven cycles and the drift period is 14 cycles (about 154-yr). The reversal points are in Cycle 9, 16 and likely in Cycle 23. The following studies support our results:

- (1) Oliver et al. (1998) suggested that new magnetic flux was ejected to the dynamo region after sunspot Cycle 16. It is in agreement with the reversal point in Cycle 16.
- (2) Wilson (1988) pointed out that the next local minimum of 8-cycle modulation should be in Cycle 22, which is close to the lower current Cycle 23. It suggests that Cycle 23 should be another reversal point. It also suggests that Cycle 24 should be stronger than Cycle 23.
- (3) Duhau (2003) pointed out that the modulation of maxima is a measure of the toroidal magnetic field strengths, and that the sunspots before Cycle 16 behaved differently from those after Cycle 16. It further indicates that Cycle 16 is a reversal point.
- (4) The 14-cycle (about 154-yr) long period, proposed by the authors, is near to 149-yr (De Meyer 1998), 154-yr (Du 2006a; Du & Du 2006), 167-yr (Cohen & Lintz 1974), 171-yr (Duhau 2003), double 78.5 yr (Cole 1973), and is almost one-half of 313-yr (De Meyer 1998) or 314-yr (William & Sonett 1985; Bracewell & William 1986; Bracewell 1986).

From linear analysis, the amplitudes of Cycles 24 and 25 are estimated to be 149.5 ± 27.6 and 144.3 ± 27.6 , respectively. The value 149.5 satisfies the condition ($R_m \geq 125$) for violations of the “G-O” rule suggested by Komitov & Bonev (2001). It further suggests that the cycle-pair 24-25 would violate the “G-O” rule.

The past predictions of amplitude with statistical methods are often lower than the observations (Li et al. 2001b; Kane 2001). Considering the long trend cycle of about 154-yr, predictions with statistical methods would be higher than the observations from Cycle 24 onward.

According to the analysis above, the following conclusions can be obtained:

- (1) There indeed exists a “three-cycle” periodicity in the relationship between maximum amplitudes and periods.
- (2) Linear fit analysis implies that there exists a 14-cycle (about 154-yr) long term cycle.
- (3) The amplitudes of cycles 24 and 25 are estimated to be 149.5 ± 27.6 and 144.3 ± 27.6 , respectively. This “Even-Odd” pair would violate the “G-O” rule.

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References

- Ahluwalia H. S., 1998, *J. Geophys. Res.*, 103, 12103
 Ahluwalia H. S., 2000, *J. Geophys. Res.*, 105, 27481
 Bracewell R. N., William G. E., 1986, *MNRAS*, 223, 457
 Bracewell R. N., 1986, *Nature*, 323, 516

- Cohen T. J., Lintz P. R., 1974, *Nature*, 250, 398
Cole T. W., 1973, *Solar Phys.*, 30, 103
De Meyer F., 1998, *Solar Phys.*, 181, 201
Du Z. L., Wang H. N., He X. T., 2006, *ChJAA*, 6(3), 338
Du Z. L., 2006a, *AJ*, accepted
Du Z. L., 2006b, *A&A*, <http://dx.doi.org/10.1051/0004-6361:20064837>
Du Z. L., Du S. Y., 2006, *Solar Phys.*, accepted
Du Z. L., 2006c, *New Astronomy*, 12, 29
Duhau S., 2003, *Solar Phys.*, 213, 203
Dicke R. H., 1978, *Nature*, 276, 676
Eddy J. A., 1976, *Science*, 192, 1189
Feynman J., 1982, *J. Geophys. Res.*, 87, 6153
Feynman J., Fougere P. F., 1984, *J. Geophys. Res.*, 89(A5), 3023
Feynman J., Gabriel S. B., 1990, *Solar Phys.*, 127, 393
Fris-Christensen E., Lassen K., 1991, *Science*, 254, 698
Gleissberg W., 1939, *Observatory*, 62, 158
Gleissberg W., 1940, *Gaz. Astron.*, 27, 15
Gleissberg W., 1971, *Solar Phys.* 21, 240
Gnevyshev W. N., Ohl A. I., 1948, *Astron. Z.*, 25, 18
Hathaway D. H., Wilson R. M., Reichmann E. J., 1994, *Solar Phys.*, 151, 177
Hathaway D. H., Wilson R. M., Reichmann E. J., 1999, *J. Geophys. Res.*, 104, 22375
Hathaway D. H., Wilson R. M., Reichmann E. J., 2002, *Solar Phys.*, 211, 357
Kane R.P., 1978, *Nature*, 274, 139
Kane R.P., 1989, *Solar Phys.*, 122, 175
Kane R.P., 1999, *Solar Phys.*, 189, 217
Kane R.P., 2001, *Solar Phys.*, 202, 359
Komitov B., Bonev B., 2001, *ApJ*, 554, L119
Landscheidt T., 1999, *Solar Phys.*, 189, 415
Lantos P., Richard O., 1998, *Solar Phys.*, 182, 231
Li K. J., Gu X. M., 1999, *A&A*, 348, L59
Li K. J., Gu X. M., Xiang F. Y., Liu X. H., 2000, *MNRAS*, 317, 897
Li K. J., Yun H. S., Gu X. M., 2001a, *ApJ*, 554, L115
Li K. J., Yun H. S., Gu X. M., 2001b, *A&A*, 368, 285
Li K. J., Zhan L. S., Wang J. X. et al., 2002, *A&A*, 392, 301
Ogurtsov M. G., Nagovitsyn YU. A., Kocharov G. E. et al., 2002, *Solar Phys.*, 211, 371
Ohl A. I., 1966, *Soln. Dann No.* 12, 84
Ohl A. I., Ohl G. I., 1979, In *NASA. Marshall Space Flight Center Solar-Terrestrial Predictions Proceedings*, 2, 258
Oliver R., Ballester J. L., Baudin F., 1998, *Nature*, 394, 552
Orfila A., Ballester J. L., Oliver R. et al., 2002, *A&A*, 386, 313
Reid G. C., 1987, *Nature*, 329, 142
Silverman S. M., 1992, *Rev. Geophys.*, 30, 333
Shastri S., 1998, *Solar Phys.*, 180, 499
Siscoe G.L., 1980, *Rev. Geophys. Space Phys.*, 18, 647
Solanki S. K., Krivova N. A., Schussler M. et al., 2002, *A&A*, 396, 1029
Thompson R. J., 1993, *Solar Phys.*, 148, 383
Usoskin I. G., Mursula K., 2003, *Solar Phys.*, 218, 319
Waldmeier M., 1939, *Astron. Mitt. Zürich*, 14, 439
Wang J. L., Han Y. H., Lin B. S. et al., 1975, *Publ. Beijing Obs.*, 5, 47
Wang J. L., 1992, *Acta Astrophys. Sin.*, 12, 369
Wang J. L., Gong J. C., Liu S. Q. et al., 2002, *ChJAA*, 2, 557
William G. E., Sonett C. P., 1985, *Nature*, 318, 523
Wilson R. M., 1988, *Solar Phys.*, 115, 397
Wilson R. M., 1990, *Solar Phys.*, 125, 143
Wilson R. M., 1992, *Solar Phys.*, 140, 181
Wilson R. M., Hathaway D. H., 1999, *J. Geophys. Res.*, 104, 2555
Wolf R., 1862, *Astron. Mitt. Zurich*, No.14
Wood K. D., 1972, *Nature*, 240, 91
Yule G. U., 1927, *Phil. Trans. R. Soc. London A*, 226, 267
Zhang G. Q., Wang H. N., 1999, *Solar Phys.*, 188, 391