

# Locally Convex Spaces and Operator Ideals

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## **Abstract**

A. Pietsch introduced the concept of  $A$ -spaces, where  $A$  denotes an operator ideal between Banach spaces.  $A$ -spaces enable us to treat many important classes of locally convex spaces in a generic way. For instance, using  $A$ -space theory we can prove several properties of nuclear-, strongly nuclear- and Schwartz-spaces in a uniform way without resorting to specific properties of the corresponding operators. Instead we only rely on generic properties of the ideals.

This dissertation explores stability of  $A$ -spaces with regards to operations on spaces. We demonstrate that, given certain pre-conditions,  $A$ -spaces exhibit some remarkable invariants. In particular we prove that sub-spaces, quotient-spaces, arbitrary topological products and countable locally convex direct sums of  $A$ -spaces inherit the properties.

In addition we prove some related theorems for the strong dual, spaces of operators and topological tensor products. We also introduce some new concepts, in particular Hom-stability and  $\otimes_a$ -stability of ideals, where  $a$  equals  $\varepsilon$  or  $\pi$ .

We conclude this dissertation by highlighting some aspects of co- $A$ -spaces and mix- $A$ -spaces.

We include several examples of the various theorems. This way we aim to convince the reader of the validity and usefulness of the theorems presented in this dissertation.