Published in: Informatiewetenschap 1998, E. de Smet (ed.). Werkgemeenschap Informatiewetenschap: Antwerp (Belgium), pp. 135-145.

Evenness as a descriptive parameter for department or faculty evaluation studies

Ronald Rousseau
Informatie & Bibliotheekwetenschap (IBW)
UIA, Universiteitsplein 1, B 2610 Wilrijk, België
E-mail: ronald.rousseau@kh.khbo.be

Abstract

We define the notion of evenness and show how the Lorenz curve can be used to compare evenness between abundance vectors. It is then illustrated how the Gini evenness measure can be used as a descriptive parameter in faculty evaluation studies.

1. Introduction: the notion of evenness

The term evenness originates from the biological sciences, more particular from ecological studies (Magurran, 1991; Nijssen et al., 1998). Magurran (1991) describes evenness roughly as 'how equally abundant species are' (p.7). More precisely, evenness is a measure for the relative apportionment of abundances among the species present. We will now translate this concept in a more general language (mathematics) applicable to scientometric studies. An N-tuple (vector) such as $(x_1,x_2,\ldots,x_N)$ will denote a collection of N sources, where the i^{th} source produces x_i items. We will follow the ecological terminology and refer to such a vector as an abundance vector. In ecology sources denote, e.g., species and items the individuals (or abundances) of each species present in the region under investigation. In informetrics sources can be authors and items the articles published by each author during a fixed time span. Other examples of source-item relations can be found e.g. in (Rousseau, 1991). In this article sources will be research groups at a university department and items the publications co-authored by members of this group. This leads to the following description:

> *Evenness is a measure for the relative apportionment of items among sources (actually present, or assumed to be possibly present).*

The sentence between parentheses refers to the fact that we allow for zero-item sources such as university professors with no publications (during a particular year).

Evenness can best be described by a partial order relation (as we will show). Recall that a relation R, defined on a set U, is a partial order if R is reflexive, transitive and antisymmetric. The class of all subsets of a fixed set U, considered with the inclusion relation is the prototype of a partially ordered set, in short: poset. Within a poset elements may not be comparable. If, however, for every x and y in U we have xRy or yRx, the order is said to be complete or total. The inclusion does not yield a total order, but the set of real numbers, |, with the natural ordering, #, is a totally ordered set.

2. The Lorenz curve

The evenness poset we present is based on the Lorenz curve. Although the construction of a Lorenz curve was explained during the first STINFON conference (Rousseau,1991) we will briefly recall how this is done. First, sources are ranked according to the number of items they produce (from low to high). Then cumulative proportions of the sources as abscissas (X) are drawn against correspondingly ranked cumulative proportions of items as ordinates (Y). Consider as an example the vector (2,5,1,2). It is first rewritten as (1/10, 2/10, 2/10, 5/10), then straight lines join the co-ordinates (0,0), (1/4,1/10), (2/4,3/10), (3/4,5/10) and (4/4,10/10). In Fig. 1, two Lorenz curves are drawn. Line B, the straight diagonal line, is the Lorenz curve of perfect evenness. This line is often taken as a reference line. If any change occurs the new Lorenz curve will fall below the line of perfect evenness. Line represents the Lorenz curve of (2,5,1,2). If abundances are ranked from low to high, one obtains a convex curve, as shown in Fig.1.

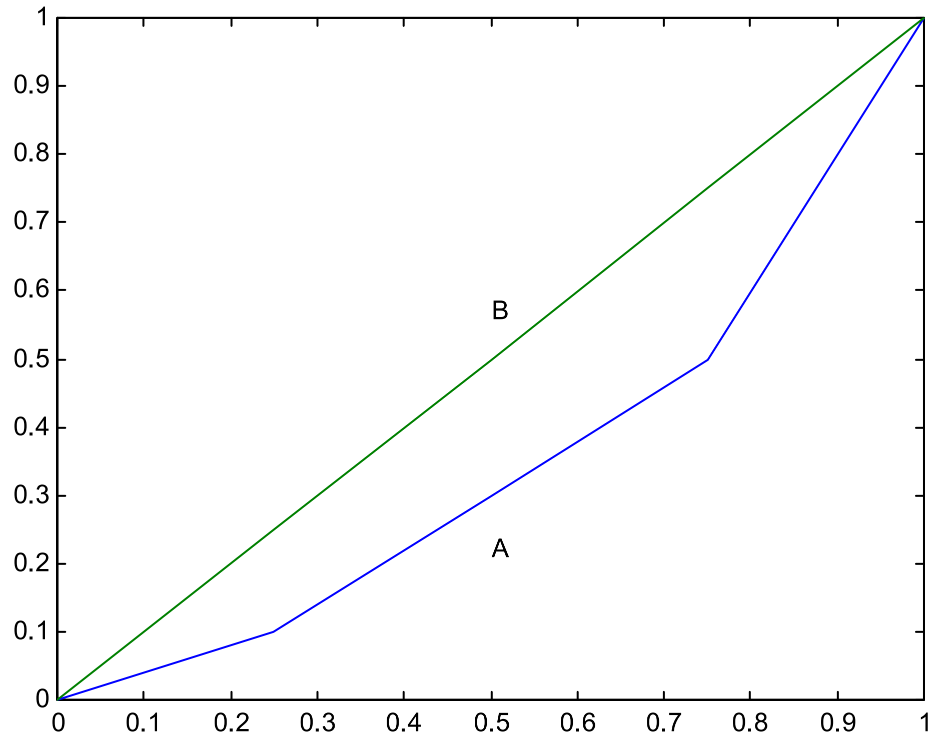

Fig.1 A: Lorenz curve for the vector (2,5,1,2)
B: Lorenz curve of perfect evenness

Lorenz curves determine a partial order in the set of vectors with the same length (N). Indeed, using the convex representation presented above we say that curve L_1 is dominated by curve L_2 if for every x-value the corresponding y-value on L_1 is smaller than or equal to the corresponding value on L_2. Graphically, this means that the curve L_1 never lies strictly above L_2. This relation will be denoted as: L_1 -< L_2. The relation -< introduces a poset structure in the set of Lorenz curves: a curve never lies above itself (L_1 -< L_1); if L_1 never lies strictly above L_2 (L_1 -< L_2), and L_2 never lies strictly above L_1 (L_2 -< L_1) then they clearly must coincide ($L_1 = L_2$), and finally, if L_1 -< L_2 and L_2 -< L_3 then L_1 -< L_3. The line of perfect equality is the largest one in this evenness poset. The ordering is only partial in the sense that two Lorenz curves that intersect are non-comparable. We will refer to this partial order as the Lorenz order.

3. Evenness as a poset

Equivalent vectors have to be introduced to transfer this poset structure to the set of all abundance vectors with the same length. Abundance vectors that differ only in the order of their components, yield the same Lorenz curve, so they are said to be equivalent. This means that evenness is not a property of individual sources, but of the investigated group as a whole (the permutation invariance property).

Evenness is, moreover, scale invariant, i.e. vectors that differ only by a proportionality factor are considered to be equivalent. So bringing vectors such as (3,4,1), (6,8,2), and (1,3,4) into one equivalence class gives a one to one correspondence between abundance vectors and Lorenz curves. To make the presentation not unnecessarily complicated, we will always identify abundance vectors with their equivalence class. Consequently, we say that vector X -< vector X' if, and only if, the Lorenz curve corresponding to X' dominates the Lorenz curve corresponding to X.

Dalton (1920) came up with another requirement: the transfer principle. Slightly adapted, this principle says that when a group that publishes only a small number of articles decreases its production further, while an already very active group increases its scientific production with the same amount, then evenness should decrease. It has been shown that the Lorenz order also meets the transfer principle (Egghe & Rousseau, 1991; Rousseau,1992).

Consequently, vectors with the same number of sources are (partially) ordered by the Lorenz order. This order coincides with the majorisation order as studied by mathematicians since the beginning of the century (see e.g. Hardy et al.,1952). In this way we have shown that evenness can be described by a – well-known – poset structure.

4. Evenness functions

In the previous section we have shown that evenness is best expressed as a partial order and that this poset structure can adequately be visualised by Lorenz curves. Historically, scientists have always tried to attach a number to an evenness situation. This procedure automatically maps the Lorenz order into a finer partial order. A mapping that realises this transition is called an evenness function. Such a function F associates a nonnegative number with each vector. Moreover, it should respect the Lorenz order, -<, i.e. if X and X' are abundance vectors and X -< X' then F(X) # F(X'). It can be shown (Rousseau,1992) that, e.g. the reciprocal of Simpson's index ($1/\lambda$), one minus Simpson's index ($1-\lambda$), the reciprocal of the coefficient of variation ($1/V = \mu/\sigma$), Shannon-Wiener's diversity (or entropy) index (H') and the Gini evenness coefficient (G') are acceptable evenness measures (for fixed N). This means that they meet the three basic requirements: permutation invariance, scale invariance and the transfer principle. The functions λ, V, H' and G' are defined in the appendix.

5. Evenness and a varying number of sources

Until now, we have kept the number of sources (N) fixed. Of course, in real situations, one wants to compare the evenness of populations with a different number of sources. To deal with this nothing has to be changed. Indeed, any abundance vector can be represented by a Lorenz curve, and any two Lorenz curves can be compared as we did in the case of a fixed number of sources. An abundance vector X over N sources is then considered to describe a more even situation than a vector X' over N' sources if the Lorenz curve of the first never lies under the Lorenz curve of the second. We will refer to this partial order on all abundance vectors as the generalised Lorenz order. We will denote it as -<< (as it is, strictly speaking, a different order relation). Using Lorenz curves and the corresponding generalised Lorenz order leads to two important advantages. They are formulated as properties.

1°) The inheritance property

This means that the restriction of the generalised Lorenz order to the case of fixed N, recovers the (usual) Lorenz order.

2°) The replication property

In the literature we find the following requirement (Dalton,1920), called the replication axiom (or property): the evenness of a population equals the evenness of any replication of that population. In other words, the evenness of (1,3,4,10) is the same as the evenness of (1,3,4,10,1,3,4,10), or of (1,3,4,10,1,3,4,10,1,3,4,10, 1,3,4,10) etc... According to permutation invariance, this is the same as the evenness of (1,1,3,3,4,4,10,10), or of (1,1,1,1,3,3,3,3,4,4,4,4,10,10,10,10). Replication clearly has no influence on the Lorenz curve.

Functions that respect the Lorenz order with fixed N and yield the same value for replicated vectors respect -<< , i.e. the generalised Lorenz order. From the evenness functions mentioned in the previous section only the Gini evenness measure G', and the reciprocal of the coefficient of variation, 1/V, respect the generalised Lorenz order. It is, however, possible to define small adaptations of the Simpson and the entropy index that do respect the generalised Lorenz order (see (Nijssen et al., 1998) for a mathematical proof of this).

6. An application: evenness of the production of a university department

Table 1 presents the number of research groups at LUC (faculty of Science and Medicine), the average number of publications and the average number of citations per group, over a thirteen-year period (1984-1996). Citations refer to publications during this period and are only counted during the first four year after publication (the year of publication being year one). This implies that the first three averages of column 4 in Table 1 are only averages over 1, resp.2 and 3 years. These averages will further not be taken into account. Data were collected as a step in a global evaluation exercise of the faculty of Sciences and Medicine of LUC (Rousseau,1998). Publication as well as citation averages per research group show a steady increase. Evenness for publications, as measured using the Gini evenness index remains roughly constant, although there is a small decline over the last five years. The reciprocal of the coefficient of variation shows more fluctuations, but declines also (slightly) over the last five years. Evenness for citations, however, shows a clear decline since 1989. Moreover, evenness for publications is always larger than evenness for citations. One way to interpret these results is that, although most groups increase their publication output and receive more citations, visibility as measured by short-term citations has become more unequal and tends to enlarge the differences between groups. Note also that evenness is larger for publications than for citations. This observation has been made before (Allison, 1980) and a possible explanation, based on positive reinforcement, has been proposed (Rousseau, 1992).

Table 1 Gini evenness values for publications and citations of LUC research groups

Year – Number of research groups	Average number of publications	Value of Gini evenness measure for publications	Average number of citations	Value of Gini evenness measure for citations
1984 – 19	2.78	0.500	(0.32)	(0.175)
1985 – 19	3.58	0.588	(1.26)	(0.241)
1986 – 19	4.00	0.600	(4.42)	(0.303)
1987 – 19	4.58	0.606	7.00	0.365
1988 – 19	5.00	0.566	9.16	0.404
1989 – 19	5.53	0.604	10.8	0.471
1990 – 20	5.40	0.586	11.4	0.418
1991 – 20	6.45	0.543	15.2	0.413
1992 – 20	5.75	0.593	22.9	0.400
1993 – 20	7.90	0.563	25.7	0.359
1994 – 21	9.67	0.521	25.4	0.342
1995 – 21	10.57	0.536	30.8	0.351
1996 – 22	11.45	0.540	36.2	0.323

Table 2 The reciprocal of the coefficient of variation for publications and citations of LUC research groups

Year – Number of research groups	Reciprocal of coefficient of variation for publications	Reciprocal of coefficient of variation for citations
1984 – 19	1.028	(0.547)
1985 – 19	1.151	(0.616)
1986 – 19	1.493	(0.672)
1987 – 19	1.342	0.749
1988 – 19	1.241	0.872
1989 – 19	1.399	0.965
1990 – 20	1.328	0.875
1991 – 20	1.050	0.813
1992 – 20	1.364	0.805
1993 – 20	1.227	0.725
1994 – 21	1.100	0.694
1995 – 21	1.046	0.706
1996 – 22	1.053	0.685

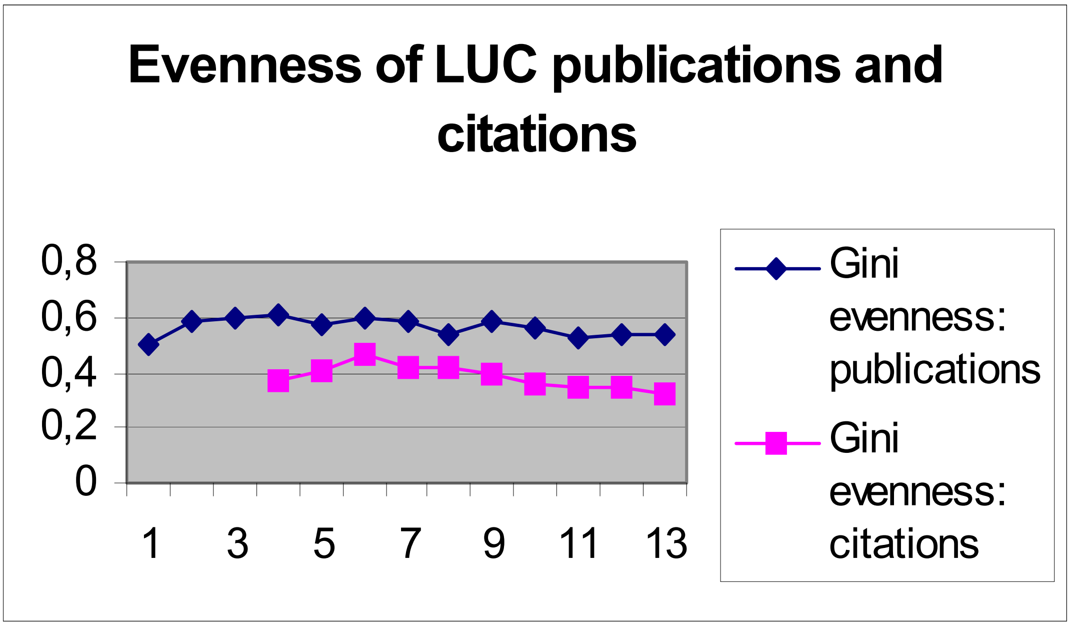

Fig.2 Evolution of Gini evenness indices over a period of 13 years

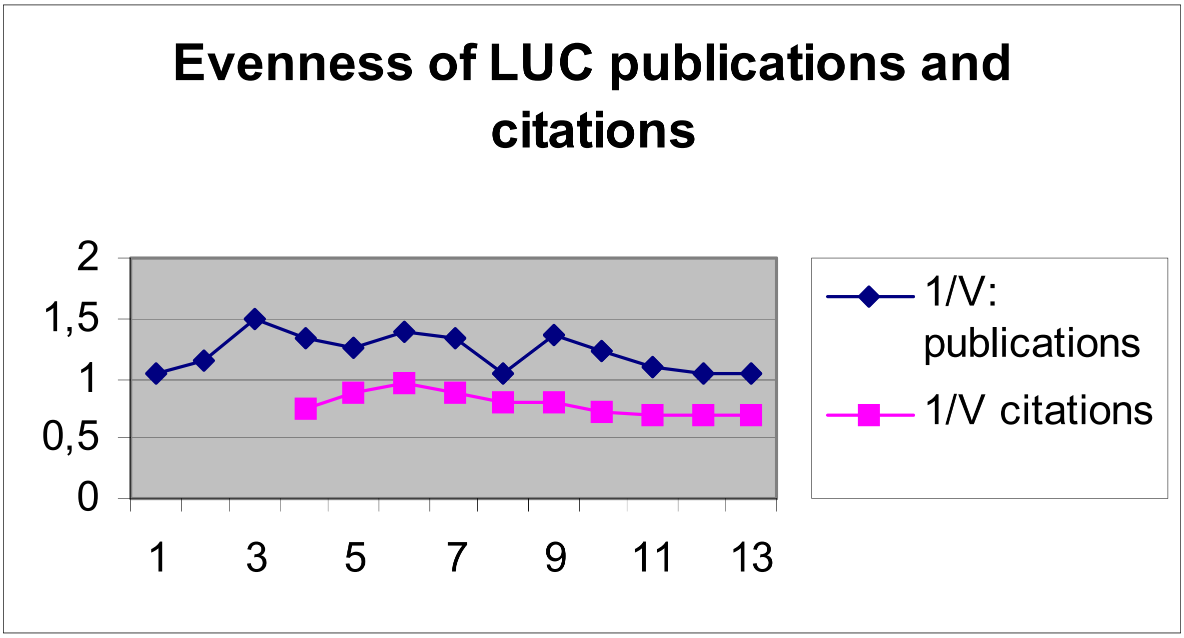

Fig.3 Evolution of the reciprocal of the coefficient of variation over a period of 13 years

7. Conclusions

Based on our investigations, cf. (Nijssen et al. 1998), we claim, with Taillie (1979), that the Lorenz curve is the best possible representation of the notion of 'evenness'. This graphical representation induces a partial order on the set of abundance vectors (more precisely: the equivalence classes of abundance vectors). If one wants to associate a number expressing the evenness of an abundance vector then the Gini evenness index G' and the reciprocal of the coefficient of variation are – probably the simplest - acceptable functions. We have shown how these measures can be used as descriptive parameters in evaluation studies. Although many reports exist evaluating the research done in the Netherlands and in Flanders (see e.g. (De Bruin et al., 1993) we are not aware of any that uses evenness as a descriptive parameter.

Acknowledgement. We thank Prof. H. Martens, rector of LUC for giving us the permission to publish data pertaining to his university. We also thank Prof. P. Van Hecke (UIA, Wilrijk, Belgium) for clarifying our views on evenness in an ecological context.

REFERENCES

Allison, P.D. (1980). Inequality and scientific productivity. *Social Studies of Science*, 10, 163-179.

Dalton, H. (1920). The measurement of the inequality of incomes*. The Economic Journal* 30, 348-361.

De Bruin, R., Kint, A., Luwel, M. and Moed, H.F. (1993). A study of research evaluation and planning: the University of Ghent. *Research Evaluation*, 3, 25- 41.

Egghe, L. and Rousseau, R. (1991). Transfer principles and a classification of concentration measures. *Journal of the American Society for Information Science*, 42, 479-489.

Hardy, G., J.E. Littlewood, and G. Pólya (1952). *Inequalities* (sec.ed.). Cambridge University Press,Cambridge,England.

Magurran, A.E. (1991). *Ecological diversity and its measurement*. Chapman & Hall, London, England.

Nijssen,D., Rousseau,R. and Van Hecke,P. (1998). The Lorenz curve: a graphical representation of evenness. *Coenoses* (to appear).

Rousseau,R. (1991). Information production processes as a general framework in informetric studies. In: *Informatiewetenschap 1991* (G. Kempen & P. de Vroomen, eds.). Leiden:STINFON, 193-201.

Rousseau,R. (1992). Concentration and diversity of availability and use in information systems: a positive reinforcement model. *Journal of the American Society for Information Science,* 43, 391-395.

Rousseau, R. (1992). *Concentration and diversity in informetric research*. Ph.D. thesis, Universitaire Instelling Antwerpen.

Rousseau,R. (1998). *A scientometric study of the scientific publications of LUC (Limburgs Universitair Centrum). Period 1984-1996.* LUC report.

Taillie,C. (1979). Species equitability: a comparative approach. In*: Ecological Diversity in Theory and Practice* (J.F. Grassle, G.P. Patil, W. Smith and C. Taillie, editors) International Co-operative Publishing House,Fairland, MD, USA, 51-65.

Appendix A

Definitions. If N denotes the number of sources then:

Simpson's index is defined as:

$$\lambda = \sum_{i=1}^{N} a_i^2 , \quad with \;\; a_i = \frac{x_i}{\sum_{j=1}^{N} x_j}$$

The coefficient of variation, denoted as V, is defined as the standard deviation, σ, divided by the mean, μ. So:

$$V = \frac{\sigma}{\mu} \;\; and \;\; hence \;\; \frac{1}{V} = \frac{\mu}{\sigma}$$

The Shannon-Wiener diversity index or entropy index is defined as

$$H'(X) = -\sum_{i=1}^{N} (a_i)\ln(a_i)$$

and finally G' is defined as:

$$G'(X) = \frac{2}{\mu N^2}\left(\sum_{i=1}^{N}(N+1-i)x_i\right) - \frac{1}{N}$$

where the x_is are ranked from low to high and μ denotes the mean of the set $\{x_i\}$. Recall that G'(X) is equal to twice the area under the Lorenz curve. Consequently, the value of the Gini evenness measure for the equality situation is equal to one.