Dynamic Probit Models and Financial Variables in Recession Forecasting

HENRI NYBERG*
Department of Economics and HECER, University of Helsinki, Finland

ABSTRACT
In this paper, various financial variables are examined as predictors of the probability of a recession in the USA and Germany. We propose a new dynamic probit model that outperforms the standard static model, giving accurate out-of-sample forecasts in both countries for the recession period that began in 2001, as well as the beginning of the recession in 2008. In accordance with previous findings, the domestic term spread proves to be an important predictive variable, but stock market returns and the foreign term spread also have predictive power in both countries. In the case of Germany, the interest rate differential between the USA and Germany is also a useful additional predictor. Copyright © 2009 John Wiley & Sons, Ltd.

KEY WORDS dynamic probit models; recession forecast; term spread; stock return

INTRODUCTION
A substantial amount of research has considered the predictive ability of various financial variables to predict the economic growth and recession periods in different countries. Much of the previous analysis is focused on time series models where the dependent variable is ‘continuous’, such as the growth rate of real GDP (see, for example, the survey by Stock and Watson, 2003). However, in the recent econometric literature, forecasting a binary recession indicator with probit or logit models has attracted attention and, consequently, new time series models for binary dependent variables have been introduced. Dueker (2002, 2005), Chauvet and Potter (2005) and Startz (2008), among others, have proposed dynamic extensions to the standard static probit model used by Estrella and Hardouvelis (1991), Bernard and Gerlach (1998), and Estrella and Mishkin (1998), among others, to predict recession periods. The main objective of this paper is to apply the dynamic models suggested by Kauppi and Saikkonen (2008) to predict monthly recession periods in the USA and Germany.

Among various financial explanatory variables considered in the paper, the term spread, which is the difference between the long-term and short-term interest rate, has proved to be a useful predictor of future economic growth and recession periods (see, for example, Estrella and Mishkin, 1998;
Estrella, 2005a). However, other financial predictors have also been suggested. For instance, if the
domestic spread is a useful predictor, then the foreign spread may also have predictive power
(Bernard and Gerlach, 1998). A potentially useful alternative is the interest rate differential between
the considered countries, which to the best of our knowledge has not been used in recession forecast-
ing prior to this paper. Furthermore, as a forward-looking variable, the stock market return should
also have additional predictive power in addition to interest rate-based predictive variables (Estrella
and Mishkin, 1998).

Our findings extend the earlier literature in several ways. We confirm that the domestic term spread
is the primary predictive variable, but we also find stock returns to have statistically significant pre-
dictive power for both countries. Furthermore, in the case of German recessions the interest rate
differential between the USA and Germany is also a useful predictor, whereas the German term
spread helps predict the US recessions. The US term spread is also a statistically significant explana-
tory variable in all predictive models fitted for German recession periods, but its out-of-sample pre-
dictive performance seems to be poor. Out-of-sample forecasts also lend some support to an
asymmetric impact of term spread on the recession probability dependent on the state of the
economy. Overall, dynamic probit models outperform the standard static recession prediction models
in terms of both in-sample and out-of-sample predictions. The best models also provide accurate
out-of-sample forecasts and recession signals for the beginning of the recession in 2008.

The paper is organized as follows. The next section presents the probit models to be used in fore-
casting, and provides a brief discussion on multiperiod forecasts of the recession indicator. In the
third section the results of the in-sample and out-of-sample predictions of recession periods in the
USA and Germany are provided. Finally, the fourth section concludes.

**DYNAMIC PROBIT MODELS**

**Models**

In binary time series analysis, the dependent variable \( y_t, t = 1, 2, \ldots, T \), is a realization of a stochastic
process that only takes on values one and zero. In recession forecasting, the value of an observable
binary recession indicator depends on the state of the economy in the following way:

\[
y_t = \begin{cases} 1, & \text{if the economy is in a recessionary state at time } t, \\ 0, & \text{if the economy is in an expansionary state at time } t \\ \end{cases}
\]  

(1)

In other words, conditional on the information set \( \Omega_{t-1} \), \( y_t \) has a Bernoulli distribution:

\[
y_t | \Omega_{t-1} \sim B(p_t)
\]  

(2)

Let \( E_{r,-1}(\cdot) \) and \( P_{r,-1}(\cdot) \) denote the conditional expectation and conditional probability given the
information set \( \Omega_{r-1} \), respectively. In the probit model the conditional probability that \( y_t \) takes the
value 1 can be written as

\[
p_t = E_{r,-1}(y_t) = P_{r,-1}(y_t = 1) = \Phi(\pi_t)
\]  

(3)

where \( \pi_t \) is a linear function of variables included in the information set \( \Omega_{r-1} \) and \( \Phi(\cdot) \) is a standard
normal cumulative distribution function.


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In the previous recession forecasting research, the standard ‘static’ model has been the most commonly used model (see, for example, Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Bernard and Gerlach, 1998). That is,

\[ \pi_t = \omega + x'_t \beta \] (4)

where \( x_{t-k} \) is the vector of explanatory variables. In \( x_{t-k} \), the \( i \)th \((i = 1, \ldots, n)\) explanatory variable, \( x_{i,t-k} \), should satisfy the condition \( k \geq h \), where \( h \) is the forecast horizon and the employed lag \( k \) may be different in different predictive variables.

A major shortcoming of the static model (4) is that it does not take the autocorrelation structure of the binary time series into account (Dueker, 1997). In recession forecasting, this means that the previous states of the economy are not included in the model. Therefore, a natural dynamic extension to the static model (4) is obtained by adding a lagged value of the dependent time series, \( y_{t-l} \), to the right-hand side of (4). This yields the ‘dynamic’ probit model\(^1\)

\[ \pi_t = \omega + \delta y_{t-l} + x'_t \beta \] (5)

where \( l \geq 1 \). Kauppi and Saikkonen (2008) extend this model by adding a lagged value of \( \pi \). The resulting ‘dynamic autoregressive’ model is given by

\[ \pi_t = \omega + \alpha \pi_{t-1} + \delta y_{t-l} + x'_t \beta \] (6)

One can also consider an ‘autoregressive’ model obtained by restricting the coefficient \( \delta \), in (6) to zero, that is,

\[ \pi_t = \omega + \alpha \pi_{t-1} + x'_t \beta \] (7)

When \( \left| \alpha \right| < 1 \), it can be seen that by recursive substitution

\[ \pi_t = \sum_{i=1}^{\infty} \alpha^{-i} \omega + \delta \sum_{i=1}^{\infty} \alpha^{-i} y_{t-i+1} + \sum_{i=1}^{\infty} \alpha^{-i} x'_{t-i+1} \beta \]

so that the dynamic autoregressive model (6) is an ‘infinite’-order extension of the dynamic model (5). This presentation indicates that the autoregressive models, (6) and (7), may be useful and parsimonious specifications if a large number of explanatory variables are helpful in forecasting. Rydberg and Shephard (2003) proposed a model somewhat similar to (6), but their model does not imply dependence on the infinite history of the explanatory variables. Throughout this paper, only one lagged value of \( \pi_t \) and of the recession indicator \( y_t \) are assumed, but including several lags is of course possible.

An interesting extension of model (7) is obtained by including an interaction term:

\[ \pi_t = \omega + \alpha \pi_{t-1} + x'_t \beta + y_{t-l} \pi_{t-l} y \] (8)

\(^1\)We use the terminology of Kauppi and Saikkonen (2008). All extensions of the static model (4) are called dynamic models, although model (5) is referred to as the ‘dynamic’ probit model.
where \( a \geq 1 \). Note that the explanatory variables included in \( z_{t-k} \) may be different from those in \( x_{t-k} \). If \( z_{t-k} = x_{t-k} \), the impact of the explanatory variables in \( x_{t-k} \) is allowed to depend on the state of the economy (cf. Kauppi and Saikkonen, 2008). Of course, it is also possible to augment model (8) by the lagged value \( y_{t-l} \), \( l \geq 1 \).

The parameters of models (4)–(8) can be estimated by the method of maximum likelihood (ML). Unfortunately, there is no formal proof of the asymptotic properties of the maximum likelihood estimator in models (6)–(8) with an autoregressive structure. However, the results of Estrella and Rodrigues (1998) and de Jong and Woutersen (2009) indicate that under reasonable regularity conditions, such as the stationarity of the explanatory variables, the ML estimator is consistent and asymptotically normal. Robust standard errors allowing for autocorrelation can be obtained as in Kauppi and Saikkonen (2008).

**Forecasts for the recession indicator**

Kauppi and Saikkonen (2008) show how one period and multiperiod forecasts in models (4)–(8) can be constructed by explicit formulae. A practical problem with recession forecasting is that realized values of the recession indicator \( y_t \), defined in (1) are known after a considerable delay. The initial announcements of many major indicators of economic activity are preliminary and often subject to substantial revision. Thus it is difficult to identify the turning points in real time. For instance, the most recent announcements of business cycle peak and trough months in the USA have taken place from 5 up to 20 months after the business cycle turning point occurred.\(^2\)

In this study, the ‘publication lag’ in the recession indicator is assumed to be 9 months. Owing to this assumed delay, the forecast horizon \( h \) consists of two periods. The first 9 months \( h = 1, 2, \ldots, 9 \), are related to predictions of the most recent past values and the current value of the recession indicator. The longer-horizon forecasts \( (h \geq 10) \) are presumably the most interesting ones. Later in this paper, this ‘ahead’ forecast horizon is denoted by \( h' \), and defined as \( h' = h - 9 \) for \( h \geq 10 \), where the number 9 is the assumed publication lag.

Kauppi and Saikkonen (2008) propose two methods of computing multiperiod recession forecasts, termed ‘direct’ and ‘iterative’ (cf. forecasts in continuous dependent time series models in, for example, Marcellino et al., 2006). A ‘direct’ forecast is obtained by employing lagged values of the dependent variable \( y_{t-l} \) and explanatory variables \( x_{t, r-k} \) when \( k \geq h' \), provided that \( k \geq 1 \) and \( l \geq h \). This forecast is direct in the sense that the right-hand side of model (6), for example, gives the \( h \)-step forecast ‘directly’. An ‘iterative’ forecast at time \( t - h \) is obtained by accounting for all possible paths and their probabilities between \( y_{t-h} \) and \( y_t \) using the same one-period model iteratively. Typically, \( y_{t-1} \) is used in the one-period model instead of forecast horizon-specific predictor \( y_{t-l} \) employed in direct forecasts.

**EMPIRICAL ANALYSIS OF RECESSION PERIODS IN THE USA AND GERMANY**

**Data and predictive variables**

In this paper, we consider the domestic and foreign term spreads (SP\(_t\)), stock market returns (\( r_t \)) and the interest rate differential between the USA and Germany (IS\(_t\)) as predictive variables in probit models. The term spread, defined as the difference between the long-term and the short-term interest rates, has been the most commonly used predictor in recession forecasting. The study of Estrella

and Hardouvelis (1991) was among the first to find the term spread a useful predictor of economic growth and recession periods in the USA. Bernard and Gerlach (1998), for instance, present similar evidence for Germany. Estrella (2005a, b) and the references therein provide an extensive literature review and the main theoretical basis for the predictive power of the term spread.

Using the static probit model (4), Bernard and Gerlach (1998) show that, in addition to the domestic term spread, foreign term spreads are also useful predictors in some considered countries. Estrella and Mishkin (1998) find that the stock return is the only variable that has out-of-sample predictive power beyond the domestic term spread to predict US recession periods in model (4). These variables have not been considered as predictors in dynamic probit models previously. Davis and Fagan (1997) include the interest rate differentials between EU countries to predict the output growth, but the evidence in favor of its predictive ability is quite weak. To the best of our knowledge, interest rate differentials between different countries have not been considered previously in recession prediction models.

The dataset includes values of the recession indicator $y_t$ and the considered explanatory variables $x_t$ in the USA and Germany covering the period from January 1972 to December 2007. We adopt the recession periods defined by the National Bureau of Economic Research (NBER) for the USA and Economic Cycle Research Institute (ECRI) for Germany. The dataset of predictive variables is collected from various sources.3

**In-sample results and model selection**

In the in-sample analysis, the sample period from January 1972 to December 1994 is used to examine the performance of different probit models with various combinations of explanatory variables. In model evaluation, the main goodness-of-fit measure is the pseudo-$R^2$ measure suggested by Estrella (1998). Values of some other statistical measures are also presented in Table I. We experiment with different lag orders $k$ and $l$ of the explanatory variables $x_{t-k}$ and the lagged dependent variable $y_{t-l}$, respectively, with $k$ and $l$ varying between 1 and 12. In practice, it has been common to set $k$ and $l$ equal to the forecast horizon $h$. On the other hand, Estrella and Mishkin (1998) and Kauppi and Saikkonen (2008) have emphasized that the latest values of the predictive variables included in the information set at the time the forecast is made are not necessarily the best ones in terms of predictive power. This indicates that better results may be obtained by employing lags supported by model selection.

Tables I and II show the estimation results of the best in-sample models for both countries.4 The sixth lags of the domestic and foreign term spreads performed consistently better, on average, than the alternative lag orders in different probit models for both countries. Based on the model selection, the best lag orders for the stock returns and the interest rate differential are also used in the estimation results presented in Tables I and II.

The main findings are very much the same for both countries. According to the model selection criteria, the first lag of the dependent variable $y_{t-1}$ is superior for both countries, and it is a highly statistically significant predictor. This is in line with the findings of Kauppi and Saikkonen (2008), and it gives tentative evidence that the iterative multiperiod forecasts could be superior to horizon-specific direct forecasts in out-of-sample forecasting.

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4 Details of all model selection results are available upon request.
The domestic term spread is the primary financial explanatory variable, but the foreign term spread and most stock return lags are also statistically significant predictors. The signs of the estimated coefficients are negative as expected, indicating an increased probability of recession when the values of the term spreads are relatively low. Negative stock returns also increase the probability of recession, and it appears that the predictive power is distributed among several preceding stock market returns.

The interest rate differential is a statistically significant predictor in the case of Germany. Its negative coefficient means that the recession probability increases when the short-term interest rate is higher in Germany than in the USA. However, in the USA the interest rate differential turned out to be statistically insignificant predictor and is therefore not included in the reported models in Table I.

Overall, based on the in-sample evidence for both countries, it is clear that the foreign term spreads, several lagged stock returns and the interest rate differential in Germany add significantly to the predictive power of a model that contains only the domestic term spread as a single explanatory variable. The dynamic models (5)–(8) outperform the static model (4) in terms of in-sample performance. However, the static model augmented with the above-mentioned additional

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<td>0.84</td>
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<td>0.97</td>
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Note: The models are estimated using monthly observations of the recession indicator (1) and explanatory variables from 1972 M1 to 1994 M12, T = 276. Robust standard errors suggested by Kauppi and Saikkonen (2008) are reported in parentheses. In the table, ps. R² reflects the pseudo-R² and adj. ps. R² the adjusted pseudo-R² (Estrella, 1998), which is calculated as

\[
\frac{1 - (1 - \text{ps. } R^2) \cdot T - K}{T - K - 1},
\]

where K is the number of estimated parameters. In addition, AIC and BIC are the values of the Akaike (1974) and Schwarz (1978) information criteria, and QPS is the quadratic probability score (Diebold and Rudebusch, 1989). Furthermore, CR50% and CR25% indicate the ratio of correct predictions with 50% and 25% threshold values in the classification of recession probabilities.
explanatory variables also outperforms the traditional static model where the domestic term spread is the only predictor (the first and the second models in Tables I and II).

The best in-sample fit for the USA is obtained from the dynamic model (5) with the $y_{t-1}$ predictor. On the other hand, the ‘pure’ autoregressive model (7) also yields good in-sample fit with a relatively large and highly statistically significant estimate of the autoregressive coefficient $\alpha_1$, indicating that the statistical improvement compared with the static model (4) is clear.

In the autoregressive interaction model (8) it seemed reasonable to use an interaction term of the form $y_{t-1}z_{t-k}$. The first lag of the dependent variable $y_{t-1}$ as such was excluded because its inclusion rendered the interaction term statistically insignificant, reducing the model to the dynamic model (5). In the models presented in Tables I and II, the sixth lag of the term spread is used in both $z_{t-k}$ and $x_{t-k}$. For both countries the estimate of the interaction term coefficient is statistically significant, suggesting that the US term spread has an asymmetric effect on recession probability, with the asymmetry depending on the state of the economy. As a matter of fact, this model yields the best in-sample fit for Germany. Interestingly, the US term spread has a stronger asymmetric effect than the domestic term spread on German recession periods. The evidence of the asymmetric effect of the term spread is in accordance with monetary policy having a similar asymmetric effect on the real economy (see, for instance, Morgan, 1993; Florio, 2004).

Table II. In-sample results from recession prediction models for Germany

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<td>0.77</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.01)</td>
<td>(0.03)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_{G}^{t-1}$</td>
<td>7.48</td>
<td>2.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_{G}^{t-6}SP_{US}^{t-6}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.65</td>
</tr>
<tr>
<td>$y_{G}^{t-6}SP_{US}^{t-6}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.48)</td>
</tr>
<tr>
<td>ps. $R^2$</td>
<td>0.69</td>
<td>0.78</td>
<td>0.97</td>
<td>0.98</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>Adj. ps. $R^2$</td>
<td>0.68</td>
<td>0.77</td>
<td>0.97</td>
<td>0.98</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>AIC</td>
<td>74.14</td>
<td>61.26</td>
<td>19.98</td>
<td>16.65</td>
<td>20.56</td>
<td>10.37</td>
</tr>
<tr>
<td>BIC</td>
<td>77.76</td>
<td>73.93</td>
<td>34.46</td>
<td>32.94</td>
<td>35.04</td>
<td>26.66</td>
</tr>
<tr>
<td>QPS</td>
<td>0.16</td>
<td>0.13</td>
<td>0.03</td>
<td>0.02</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>CR50%</td>
<td>0.71</td>
<td>0.90</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>CR25%</td>
<td>0.70</td>
<td>0.89</td>
<td>0.97</td>
<td>0.99</td>
<td>0.97</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: See note to Table I.
An important issue in specifying a recession forecasting model is the stability of the relationship between the explanatory variables and the recession indicator (1). We examined this by using the LM test proposed by Andrews and Fair (1988) and applied by Kauppi (2008) in the context of probit model when testing potential structural break dates suggested by Estrella et al. (2003). We found no evidence of structural breaks in the models presented in Tables I and II at conventional significance levels. The evidence is in line with the findings of Estrella et al. (2003), Wright (2006) and Kauppi (2008).

Figure 1 depicts the in-sample recession probabilities of the static model (4) and the autoregressive model (7) (the first and the fifth models in Tables I and II). These models are used as such also in out-of-sample forecasting in the following section. It can be seen that in the autoregressive model the recession probability matches better with the realized values of the recession indicator than in the static model. In recession periods, the recession probabilities are also higher in the autoregressive model. When the economy is in an expansionary state, the recession probability is constantly higher in the static model, whereas in the autoregressive model it is very close to zero, as it should be.

Out-of-sample forecasting results

The in-sample evidence shows a great deal of predictability for recession periods in the USA and Germany. However, in-sample predictability does not necessarily mean out-of-sample predictability. For instance, some of the static probit models considered by Estrella and Mishkin (1998) for the US recession periods provide the best in-sample fitted values, but perform quite poorly out of sample.

In this study, the first out-of-sample predictions are made for January 1995 and last ones for March 2007. The forecast period thus contains the recession period that began in both countries in 2001. In other months, both economies are in an expansionary state. The parameters are estimated recursively. In other words, after adding one month to the previous estimation period and re-estimating the parameters, forecasts for the next month are computed. This procedure is repeated recursively until the end of the forecast period.

We examine the out-of-sample predictive ability of the models that turned out to be the best ones according to the in-sample results presented in Tables I and II. Therefore, we employ the following explanatory variables:

\[ x_{t-k}^{US} = (SP_{t-k}, SP^{GE}_{t-k}, r_{t-k}^{US}, r_{t-k}^{US}, r_{t-k}^{US})' \]  

and

\[ x_{t-k}^{GE} = (SP_{t-k}, SP^{US}_{t-k}, r_{t-k}^{GE}, r_{t-k}^{GE}, r_{t-k}^{GE}, IS_{t-k})' \]  

Models where the foreign term spread is excluded are also examined. In these cases, the vectors of the explanatory variables are denoted by \( x_{t-k}^{US} \) and \( x_{t-k}^{GE} \). When the domestic term spread is the only predictor, the corresponding vectors are denoted by \( u_{t-k}^{US} \) and \( u_{t-k}^{GE} \).

With the forecast horizon \( h \), the lags in the explanatory variables should be tailored so that only the information included in the information set \( \Omega_{t-h} \) at the forecast time \( t-h \) is used. For example, if the forecast horizon is 16 months (\( h = 16 \)), and because the publication lag is assumed to be 9

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5 Further details are available upon request.
6 This is not the case in the models employing \( y_{t-1} \) as a predictive variable because those one-period models are used iteratively in out-of-sample forecasting to obtain the iterative recession forecast.
7 The lag orders used in the explanatory variables in these two cases are the same as in the vectors \( x_{t-k}^{US} \) and \( x_{t-k}^{GE} \) presented in (9) and (10).
months, it means that we are interested in forecasting the value of the US recession indicator 7 months ($h^f = 7$) ahead, and the vector $x_{t-k}^U$ is given by

$$x_{t-k}^U = (SP_{t-7}^U, SP_{t-7}^{GE}, r_{t-7})'$$

Forecast accuracy is evaluated by using the same statistical goodness-of-fit measures as in the in-sample analysis. In addition, 50% and 25% threshold values are used to classify recession probabilities and to construct ‘strong’, ‘weak’ and ‘no’ recession signals. For example, if the recession probability is between 25% and 50%, the model gives a ‘weak’ recession signal. Related to these signal forecasts an asymmetric ‘forecasting point’ scheme is applied. The idea, illustrated in Table

Figure 1. In-sample recession probabilities implied by the static model (4), with the domestic term spread as the only predictor and the autoregressive model (7) with additional explanatory variables for the USA (upper panel) and Germany (lower panel) as given in Tables I and II.
IV, is to put greater emphasis on correct forecasts (cf. Dueker, 2002). It also favors a false recession alarm compared with a missed recession month. One rationale behind this is that firms or policy makers, for example, are possibly willing to take a ‘recession insurance’ and accept a possible false alarm rather than be caught by an unexpected recession.

As discussed earlier in the context of multiperiod forecasting, the most interesting forecasts are typically the future values of the recession indicator. Thus we concentrate on forecasts with forecast horizon \( h \geq 10 \) \((h' \geq 1)\). The results from shorter horizons \( (h \leq 9)\) are available upon request. It is worth noting that, in practice, iterative forecasts are computationally very demanding when the forecast horizon is as long as 21 months in Tables III and IV.\(^8\) This is a difficulty of the iterative forecasting approach employed in the dynamic models (5) and (8) with \( y_{t-1} \) as a predictor. Therefore, only forecasts based on the static model (4) and the autoregressive model (7) are considered when the forecast horizon is so long that iterative forecasting becomes computationally infeasible.

Based on the adjusted-pseudo-\(R^2\) the best predictive models yield good out-of-sample forecasts for the state of the US economy. In Table III the highest values of the adjusted-pseudo-\(R^2\) in different probit models are obtained with the forecast horizon of 15 months \((h' = 6)\). At this forecast horizon, the autoregressive model (7) and the autoregressive model with the interaction term (8) outperform the dynamic

\[ hf \]

\[ hf \]

**Table III. Adjusted pseudo-\(R^2\) measures of out-of-sample predictive performance for different models in the USA**

<table>
<thead>
<tr>
<th>Model</th>
<th>( \ h )</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static (4); ( x_{t-k}^{US} )</td>
<td>0.31</td>
<td>0.31</td>
<td>0.26</td>
<td>0.29</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.27</td>
</tr>
<tr>
<td>Dyn. iter. (5); ( x_{t-k}^{US} )</td>
<td>0.50</td>
<td>0.52</td>
<td>0.49</td>
<td>0.49</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>Auto. (7); ( x_{t-k}^{US} )</td>
<td>0.46</td>
<td>0.45</td>
<td>0.46</td>
<td>0.46</td>
<td>0.45</td>
<td>0.45</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>Auto. int. (8); ( x_{t-k}^{US}, S_{t-k}^{US} )</td>
<td>0.33</td>
<td>0.30</td>
<td>0.51</td>
<td>0.51</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
</tr>
<tr>
<td>Auto. int. (8); ( x_{t-k}^{US}, S_{t-k}^{US} )</td>
<td>0.10</td>
<td>0.05</td>
<td>0.45</td>
<td>0.44</td>
<td>0.53</td>
<td>0.53</td>
<td>0.54</td>
<td>0.54</td>
<td>0.54</td>
<td>0.54</td>
<td>0.54</td>
<td>0.54</td>
<td>0.54</td>
</tr>
<tr>
<td>Static (4); ( x_{t-k}^{US} )</td>
<td>0.18</td>
<td>0.18</td>
<td>0.16</td>
<td>0.16</td>
<td>0.10</td>
<td>0.10</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>Dyn. iter. (5); ( x_{t-k}^{US} )</td>
<td>0.43</td>
<td>0.45</td>
<td>0.41</td>
<td>0.40</td>
<td>0.26</td>
<td>0.26</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Auto. (7); ( x_{t-k}^{US} )</td>
<td>0.42</td>
<td>0.42</td>
<td>0.39</td>
<td>0.39</td>
<td>0.38</td>
<td>0.38</td>
<td>0.34</td>
<td>0.34</td>
<td>0.34</td>
<td>0.34</td>
<td>0.34</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td>Auto. int. (8); ( x_{t-k}^{US}, S_{t-k}^{US} )</td>
<td>0.21</td>
<td>0.21</td>
<td>0.20</td>
<td>0.20</td>
<td>0.38</td>
<td>0.38</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>Static (4); ( u_{t-k}^{US} )</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
<td>0.08</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>Dyn. iter. (5); ( u_{t-k}^{US} )</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>Auto. (7); ( u_{t-k}^{US} )</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>Auto. int. (8); ( u_{t-k}^{US}, S_{t-k}^{US} )</td>
<td>0.31</td>
<td>0.30</td>
<td>0.29</td>
<td>0.29</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
</tr>
</tbody>
</table>

*Note*: The table presents the adjusted pseudo-\(R^2\) values (see Table I and Estrella, 1998) of different models in out-of-sample predictions. The probit model is denoted at the left, with the explanatory variables included in the model. As in (9), and in the subsequent discussion, \( x_{t-k}^{US} \) include all explanatory variables, in \( x_{t-k}^{US} \) the German term spread is excluded and in \( u_{t-k}^{US} \) only the US term spread is employed in the model. In the autoregressive interaction model (8), the term spread that is used in the interaction term \( x_{t-k}^{US} \) is also mentioned. In dynamic model (5) and in autoregressive interaction model (8) the first lagged value of the recession indicator \( y_{t-1} \) is used in the model.

\(^8\)In iterative forecasts with \( h = 21 \), \( 2^h \) different paths need to be computed and the computational burden increases rapidly if longer publication lags are considered.

\(^9\)The values of the adjusted-pseudo-\(R^2\) are adjusted to the number of parameters estimated. The evidence from different goodness-of-fit measures, such as forecasting points, information criteria or QPS, is the same. Further, and also in the case of Germany in Table IV, results from the dynamic model (5) with \( y_{t-1} \) and the dynamic autoregressive model (6) are excluded because forecasts from the restricted static model (4) and the dynamic model (5) with \( y_{t-1} \) yield almost the same or even better predictions than these more general models.
model (5), yielding the best out-of-sample forecasts given the explanatory variables included in the vector $x_{U_t-k}^E$ in (9). In fact, the autoregressive interaction model (8) is the best model when the forecast horizon is between 12 and 16 months, providing evidence that the asymmetric predictive power of the US term spread found in the in-sample analysis also shows up in out-of-sample predictions.

Overall, the models with the US stock return ($x_{U_t-k}^G$*) and the models that also include the German term spread ($x_{G_t-k}^E$, $SP_{t-6}^G$) outperform the models with the US term spread ($\upsilon_{U_t-k}^E$) as the only predictor across all probit model specifications and forecast horizons. This suggests that these additional financial variables have not only in-sample but also out-of-sample predictive content for the US recessions.

In Germany, the recession in the out-of-sample period lasted considerably longer than in the USA, but the essential conclusions between different predictive models are parallel to those for the USA. However, because the term spreads soared immediately after the recession began, the recession probability decreased amidst the recession period. Consequently, the negative values of the adjusted pseudo-$R^2$ were obtained for some models, making comparisons difficult. Therefore, the forecasting points presented in Table IV are the main model evaluation measure for Germany.

The results of Table IV confirm the previous in-sample findings. Even out of sample, the interest rate differential between the USA and Germany and the German stock return clearly have additional predictive power beyond the German term spread in all probit models. As in the case of the USA, when the forecast horizon increases towards 15 or 16 months the models with an autoregressive
model structure, (7) and (8), seem to outperform their competitors. Interestingly, the US term spread, which is a statistically significant predictor in sample, seems to be a rather poor predictor out of sample, for the forecasting results are much better without it. The statistical significance of the interest rate differential, however, suggests that the US monetary policy has an impact on the probability of recession in Germany via the US short-term interest rate.

Paap et al. (2009) propose a model that allows for asymmetries such that, for example, the term spread has a different lead time in recession and expansion. This issue can be examined with model (8) by selecting different lag orders of explanatory variables included in $z_{t-k}$ and in $x_{t-k}$. In Tables III and IV two models are considered where the predictive lag of the term spread in $z_{t-k}$ is selected based on the in-sample model selection. For instance, the selection $z_{t-k} = SPU_t - 8$ seems to be the best one for the USA. However, according to the out-of-sample results of both countries, differences between the presented two versions of model (8) using different lag orders in $z_{t-k}$ are minor.

For both countries, the best autoregressive interaction model (8) seems to generate somewhat better out-of-sample forecasts than the dynamic model (5). This is in contrast to the findings of Kauppi and Saikkonen (2008) and may be due to the additional financial explanatory variables used in this paper. However, it should be pointed out that the autoregressive model (7) produces almost as good out-of-sample predictions as model (8) for both countries. Moreover, forecasts obtained with model (7) do not require computationally intensive iterative methods. When the forecast horizon is 21 months, which is the longest horizon considered, the static model (4) without any dynamics turns out to be an adequate model. However, also with this forecast horizon, the additional explanatory variables have useful predictive power.

Figure 2 illustrates the out-of-sample performance of the static model (4) with only the domestic term spread and the autoregressive interaction model (8) with additional explanatory variables. The forecast horizon is 15 months ($h = 15$). The performance of the static model considered in many previous studies is inferior to the autoregressive interaction model for both countries. The latter model has predictive power, especially at the beginning and end of the latest recession period for both countries.

In recession forecasting the probability of continued expansion (a time period where the economy is in an expansionary state every month) is of particular interest (see Chauvet and Potter, 2005). Continued expansion probabilities give a similar impression of expansionary and recessionary periods as the month-to-month predictions discussed above. The predictive ability of the different models appears to depend on the state of the economy. During expansion the static model seems to overpredict the recession probability, whereas the dynamic models perform better in this respect (see, for example, Figure 2). On the other hand, in recession periods the dynamic model (5) constantly gives the highest probabilities of continued expansion. Thus also according to the continued expansion probabilities, the autoregressive probit models (7) and (8), including the domestic term spread and other explanatory variables, seem to yield the most reliable predictions.

Recession probabilities in 2006–2008

In this section, we consider out-of-sample recession forecasts up to June 2008. Figure 3 depicts the recession probabilities from the beginning of year 2006 for both countries. The forecast horizon is again 15 months ($h = 15$, $h' = 6$), indicating that the latest forecasts for June 2008 are based on predictive information from December 2007. In December 2008 the NBER announced that a peak in US economic activity had occurred in December 2007. Similarly, ECRI made an announcement that

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10 The results are available upon request.
for Germany a peak had occurred in April 2008. After these peak months, both countries have been in a recession.

As seen in Figure 3, the autoregressive interaction model (8) predicts the beginning of the recession well for both countries, especially for Germany. For the USA, the recession forecast exceeded the 50% threshold value in August 2007 and thereafter the recession probability has been relatively high. The static model (4) with the domestic term spread as the only explanatory variable, which has been the standard recession prediction model, does not give as precise recession and expansion signals as the autoregressive interaction model. As in the previous section, the performance of this standard model appears disappointing in comparison with different dynamic models, such as model (8).
CONCLUSIONS

In this paper, we examine the performance of recession prediction models that include a number of financial explanatory variables. The results indicate that, compared with the standard static recession prediction model used in many previous studies, statistically significant additional predictive power is obtained by allowing for dynamic structures in the model. In particular, models with an autoregressive structure outperformed the static model and they were also somewhat better than other dynamic models considered in terms of out-of-sample performance. The best model for the USA and Germany turned out to be an autoregressive interaction model in which the term spread between...
the long-term and short-term interest rate has an asymmetric effect on recession probability, with the asymmetry depending on the state of the economy.

In accordance with previous studies, the term spread is found to be a useful predictor for both the US and German recession periods, but for both countries additional predictive power is provided by stock returns. For Germany, the short-term interest rate differential between the USA and Germany also has substantial predictive power in both in-sample and out-of-sample prediction. The same holds for the German term spread when forecasting the US recessions. Furthermore, the US term spread is a statistically significant predictor in the case of Germany, but its out-of-sample predictive power appears poor.

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REFERENCES


**Author’s biography:**

**Henri Nyberg** is a Ph.D. student at the University of Helsinki. His research interests are in econometrics, time series analysis and empirical macroeconomics.

**Author’s address:**

**Henri Nyberg**, Department of Economics, PO Box 17, (Arkadiankatu 7), 00014 University of Helsinki, Finland.